# English Grammar and Constituency Parsing 

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Some slides are based on class materials from Ralph Grishman, Thien Huu Nguyen, David Bamman, Dan Jurafsky, James Martin, Michael Collins

## Syntax

With syntax, we're moving from labels for discrete items - documents (sentiment analysis), tokens (POS tagging, NER) - to the structure between items.

Syntax is fundamentally about the hierarchical structure of language and (in some theories) which sentences are grammatical in a language
words $\rightarrow$ phrases $\rightarrow$ clauses $\rightarrow$ sentences


I shot an elephant in my pajamas


Nominal
elephant


## Why Is Syntax Important?

Foundation for semantic analysis (on many levels of representation: semantic roles, compositional semantics, frame semantics)

Humans communicate complex ideas by composing words together into bigger units to convey complex meanings


## Why Is Syntax Important?

Linguistic typology; relative positions of subjects (S), objects ( O ) and verbs (V)

| SVO | English, Mandarin | I grabbed the chair |
| :---: | :---: | :---: |
| SOV | Latin, Japanese | I the chair grabbed |
| VSO | Hawaiian | Grabbed I the chair |
| OSV | Yoda | Patience you must have |
| $\ldots$ | $\ldots$ | $\ldots$ |

## Why Is Syntax Important?

Strong representation for discourse analysis (e.g., coreference resolution)

https://en.wikipedia.org/wiki/Discourse_analysis

## Formalisms



Phrase structure grammar (Chomsky 1957)


Dependency grammar
(Mel'čuk 1988; Tesnière 1959; Pāṇini)

## Constituency

Groups of words ("constituents") behave as single units
"Behave" = show up in the same distributional environments as single units (e.g., the substitution test)

Substitution test for POS: if a word is replaced by another word, does the sentence remain grammatical?

Substitution test for Constituency: if a constituent is replaced by another constituent of the same type, does the sentence remain grammatical?

## Context-Free Grammar (CFG)

A CFG gives a formal way to define what meaningful constituents are and exactly how a constituent is formed out of other constituents (or words). It defines valid structure in a language (i.e., defining how symbols in a language combine to form valid structures)


NP $\rightarrow$ Det Nominal


NP $\rightarrow$ Verb Nominal

## Context-Free Grammar (CFG)

| $N$ | Finite set of non-terminal symbols | $\mathrm{NP}, \mathrm{VP}, \mathrm{S}$ |
| :--- | :---: | :--- |
| $\Sigma$ | Finite alphabet of terminal symbols | the, dog, eat |
| $R$ | Set of production rules, each of the <br> form $A \rightarrow \beta, \beta \in(\Sigma \cup N) *$ | $\mathrm{S} \rightarrow \mathrm{NP} \mathrm{VP}$ <br> Noun $\rightarrow$ dog |
| $S$ | A designated start symbol |  |

## Derivation

Given a CFG, a derivation is the sequence of productions used to generate a string of words/terminal symbols (e.g., a sentence), often visualized as a parse tree.

$$
\text { NP VP } \rightarrow \text { cats VP } \rightarrow \text { cats chase NP }
$$


the flight

a flight

the morning flight

## Language

The strings of words (e.g., sentences) are called as "derivable from the start symbol (S)"

The formal language defined by a CFG is the set of strings derivable from S
$\mathrm{S} \rightarrow \mathrm{NPVP} \rightarrow$ cats $\mathrm{VP} \rightarrow$ cats chase $\mathrm{NP} \rightarrow$ cats chase mice

## Preterminals

It is convenient to include a set of symbols called preterminals (corresponding to the parts of speech) which can be directly rewritten as terminals (words)

This allows us to separate the productions into a set which generates sequences of preterminals (the "grammar") and those which rewrite the preterminals as terminals (the "dictionary")

## Grouping Alternates

To make the grammar more compact, we group productions with the same left-hand side:

$\mathrm{S} \rightarrow \mathrm{NP}$ VP<br>$\mathrm{NP} \rightarrow \mathrm{N} \mid$ ART $\mathrm{N} \mid$ ART ADJ N<br>$\mathrm{VP} \rightarrow \mathrm{V} \mid \mathrm{VNP}$

## Example

$$
\begin{aligned}
\text { Noun } \rightarrow & \text { flights } \mid \text { breeze } \mid \text { trip } \mid \text { morning } \\
\text { Verb } \rightarrow & \text { is } \mid \text { prefer } \mid \text { like } \mid \text { need } \mid \text { want } \mid \text { fly } \\
\text { Adjective } \rightarrow & \text { cheapest } \mid \text { non-stop } \mid \text { first } \mid \text { latest } \\
& \mid \text { other } \mid \text { direct } \\
\text { Pronoun } \rightarrow & \text { me }|I| \text { you } \mid \text { it } \\
\text { Proper-Noun } \rightarrow & \text { Alaska } \mid \text { Baltimore } \mid \text { Los Angeles } \\
& \mid \text { Chicago } \mid \text { United } \mid \text { American } \\
\text { Determiner } \rightarrow & \text { the } \mid \text { a } \mid \text { an } \mid \text { this } \mid \text { these } \mid \text { that } \\
\text { Preposition } \rightarrow & \text { from } \mid \text { to } \mid \text { on } \mid \text { near } \\
\text { Conjunction } \rightarrow & \text { and } \mid \text { or } \mid \text { but }
\end{aligned}
$$

Figure 12.2 The lexicon for $\mathscr{L}_{0}$.

| Grammar Rules |  | Examples |
| :---: | :---: | :---: |
| $S \rightarrow$ | $N P$ VP | I + want a morning flight |
| $N P \rightarrow$ | Pronoun | I |
|  | Proper-Noun | Los Angeles |
|  | Det Nominal | a + flight |
| Nominal $\rightarrow$ | Nominal Noun | morning + flight |
|  | Noun | flights |
| $V P \rightarrow$ | Verb | do |
|  | Verb NP | want + a flight |
|  | Verb NP PP | leave + Boston + in the morning |
|  | Verb PP | leaving + on Thursday |
| $P P \rightarrow$ | Preposition NP | from + Los Angeles |

Figure 12.3 The grammar for $\mathscr{L}_{0}$, with example phrases for each rule.

## Bracketed Notation


$\left[_{N P}[\right.$ Det the $]\left[\left[_{\text {Nominal }}\left[\left[_{\text {Noun }} f l i g h t\right]\right]\right]\right.$

## Constituents

## Every internal node is a phrase

- my pajamas
- in my pajamas
- elephant in my pajamas
- an elephant in my pajamas
- shot an elephant in my pajamas
- I shot an elephant in my pajamas

Each phrase could be replaced by another of the same type of constituent


## Sentence

| Rule | Description | Example |
| :--- | :--- | :--- |
| S $\rightarrow$ VP | Imperative | • Show me the right way |
| S $\rightarrow$ VP NP | Declarative | • The dog barks |
| S $\rightarrow$ Aux VP NP | Yes/no <br> questions | • Will you show me the right way? |
| S $\rightarrow$ Wh-NP VP <br> $S \rightarrow$ Wh-NP Aux NP VP | wh- <br> questions | • What airlines fly from Burbank to Denver? <br> • What flights do you have from Burbank to <br> Tacoma Washington? |

## Noun Phrases

## NP $\rightarrow$ Pronoun | Proper-noun | Det Nominal

Nominal $\rightarrow$ Nominal PP

- An elephant [pp in my pajamas]
- The cat [ ${ }_{p p}$ on the floor] [ ${ }_{p p}$ under the table] [ ${ }_{p p}$ next to the dog]

Nominal $\rightarrow$ RelClause, RelClause $\rightarrow$ (who|that) VP : A relative pronoun (that, which) in a relative clause can be the subject or object of the embedded verb.

- A flight [Relclause that serves breakfast]
- A flight [Relclause that I got]


## Verb Phrases

| $V P \rightarrow$ Verb | disappear |
| :--- | :--- |
| $V P \rightarrow$ Verb NP | prefer a morning flight |
| $V P \rightarrow$ Verb NP PP | prefer a morning flight on Monday |
| $V P \rightarrow$ Verb PP | leave on Wednesday |
| $V P \rightarrow$ Verb S | I think [S I want a new flight] |
| $V P \rightarrow$ Verb VP | want [ to fly today] |

## Verb Phrases

| $V P \rightarrow$ Verb | * I filled |
| :--- | :--- |
| $V P \rightarrow$ Verb NP | * I exist the morning flight |
| $V P \rightarrow$ Verb NP PP | * I exist the morning flight on Monday |
| $V P \rightarrow$ Verb PP | * I filled on Wednesday |
| $V P \rightarrow$ Verb S I exist [S I want a new flight] |  |
| $V P \rightarrow$ Verb VP | * I fill [ to fly today] |

## Subcategorization

Verbs are compatible with different complements

- Transitive verbs take direct object NP ("I filled the tank")
- Intransitive verbs don't ("I exist")

The set of possible complements of a verb is its subcategorization frame.

$$
\begin{array}{lll}
\text { VP } & \rightarrow \text { Verb VP } & \text { * I fill [yp to fly today] } \\
\text { VP } & \rightarrow \text { Verb VP } & \text { I want [vp to fly today] }
\end{array}
$$

## Coordination

| NP $\rightarrow$ NP and NP | the dogs and the cats |
| :--- | :--- |
| Nominal $\rightarrow$ Nominal and <br> Nominal | dogs and cats |
| VP $\rightarrow$ VP and VP | I came and saw and conquered |
| $\mathrm{JJ} \rightarrow$ JJ and JJ | beautiful and red |
| $\mathrm{S} \rightarrow$ S and S | I came and I saw and I conquered |

## Ambiguity

Most sentences will have more than one parse

## Generally different parses will reflect different meanings ...

- Attachment ambiguity: a particular constituent can be attached to the parse tree at more than one place "I saw the man with a telescope."
Can attach PP ("with a telescope") under NP or VP
- Coordination ambiguity: different sets of phrases can be conjoined by a conjunction like "and":
"old man and woman" -> [old [men and women]] or [[old man] and [woman]]?


## An Example

I shot an elephant in my pajamas
\(\left.\begin{array}{rl}\hline S \& \rightarrow NP VP <br>
VP \& \rightarrow Verb NP <br>
VP \& \rightarrow VP PP <br>
Nominal \& \rightarrow Nominal PP <br>
Nominal \& \rightarrow Noun <br>
Nominal \& \rightarrow Pronoun <br>
PP \& \rightarrow Prep NP <br>
NP \& \rightarrow Det Nominal <br>
NP \& \rightarrow Nominal <br>
NP \& \rightarrow PossPronoun <br>

Nominal\end{array}\right]\)| Verb | $\rightarrow$ shot |
| ---: | :--- |
| Det | $\rightarrow$ an $\mid$ my |
| Noun | $\rightarrow$pajamas <br> elephant |
| Pronoun | $\rightarrow$ I |
| PossPronoun | $\rightarrow$ my |




## Evaluation

Parseval (1991): represent each tree as a collection of tuples.

Calculate precision, recall, F1 from these collections of tuples

$$
<l_{1}, i_{1}, j_{1}>, \ldots,<l_{n}, i_{n}, j_{n}>
$$

- $l_{k}$ : label for the $k$-th phrase
- $i_{k}$ : index for the first word in the $k$-th phrase
- $j_{k}$ : index for the last word in the $k$-th phrase

$$
\begin{aligned}
& \bullet<S, 1,7> \\
& \bullet<N P, 1,1> \\
& \bullet<V P, 2,7> \\
& \bullet<V P, 2,4> \\
& \bullet<N P, 3,4> \\
& \bullet<\text { Nominal, 4, 4> } \\
& \bullet<P P, 5,7> \\
& \bullet<N P, 6,7>
\end{aligned}
$$



## Evaluation

- Precision $(P)=$ number of tuples in the predicted tree also in correct tree, divided by number of tuples in the predicted tree $=5 / 7$
- Recall $(R)=$ number of tuples in the predicted tree also in correct tree, divided by number of tuples in the correct tree $=5 / 7$
- $F 1=\frac{2 P R}{P+R}$
- <S, 1, 7>
- <NP, 1,1>
$\bullet<V P, 2,7>$
- <NP, 3, 7>
- <Nominal, 4, 7>
-<Nominal, 4, 4>
- <PP, 5, 7>
- <NP, 6, 7>
$\bullet<S, 1,7>$
- <NP, 1,1>
-<VP, 2, 7>
- <VP, 2, 4>
-<NP, 3, 4>
-<Nominal, 4, 4>
- <PP, 5, 7>
-<NP, 6, 7>



## Evaluation

Nonetheless, phrasal constituents are not always an appropriate unit for parser evaluation.

- In lexically-oriented grammars, such as CCG and LFG, the ultimate goal is to extract the appropriate predicate-argument relations or grammatical dependencies, rather than a specific derivation.
- We can use alternative evaluation metrics based on the precision and recall of labeled dependencies whose labels indicate the grammatical relations (Lin 1995, Carroll et al. 1998, Collins et al. 1999).

Why not measuring how many sentences are parsed correctly, instead of measuring component accuracy in the form of constituents or dependencies?

- The later gives us a more fine-grained metric
- Sentences can be long
- Distinguish between a parse that got most of the parts wrong and one that just got one part wrong


## CFGs

## Building a CFG by hand is really hard

To capture all (and only) grammatical sentences, need to exponentially increase the number of categories (e.g., detailed subcategorization info)

| Verb-with-no-complement | $\rightarrow$ | disappear |
| ---: | :--- | :--- |
| Verb-with-S-complement | $\rightarrow$ | said |
| VP | $\rightarrow$ | Verb-with-no-complement |
| VP | $\rightarrow$ | Verb-with-S-complement S |

## Treebanks

Rather than create the rules by hand, we can annotate sentences with their syntactic structure and then extract the rules from the annotations

Treebanks: collections of sentences annotated with syntactic structure (e.g., Penn Treebank)

## Penn Treebank



```
NP }->\mathrm{ DT JJ NN
NP }->\mathrm{ DT JJ NNS
NP }->\mathrm{ DT JJ NN NN
NP }->\mathrm{ DT JJ JJ NN
NP }->\mathrm{ DT JJ CD NNS
NP }->\mathrm{ RB DT JJ NN NN
NP }->\mathrm{ RB DT JJ JJ NNS
NP }->\mathrm{ DT JJ JJ NNP NNS
NP }->\mathrm{ DT NNP NNP NNP NNP JJ NN
NP }->\mathrm{ DT JJ NNP CC JJ JJ NN NNS
NP }->\mathrm{ RB DT JJS NN NN SBAR
NP }->\mathrm{ DT VBG JJ NNP NNP CC NNP
NP }->\mathrm{ DT JJ NNS , NNS CC NN NNS NN
NP }->\mathrm{ DT JJ JJ VBG NN NNP NNP FW NNP
NP }->\mathrm{ NP JJ , JJ '، SBAR '' NNS
```

| NP | $\rightarrow$ | NNP NNP |
| ---: | :--- | :--- |
| NP-SBJ | $\rightarrow$ | NP, ADJP, |
| $S$ | $\rightarrow$ | NP-SBJ VP |
| VP | $\rightarrow$ | VB NP PP-CLR NP-TMP |

## How To Parse?

Given a CFG and a sentence, how can we obtain the parse tree(s) for the sentence?

- Top-down parsing: repeat
- expand leftmost non-terminal using first production (save any alternative productions on backtrack stack)
- if we have matched entire sentence, quit (success)
- if we have generated a terminal which doesn't match sentence, pop choice point from stack (if stack is empty, quit (failure))
- Bottom-up parsing
- Inefficiency:
- the top-down parsers waste effort to explore trees that are not consistent with the input while
- the bottom-up parsers waste effort to explore trees that cannot lead to the start symbol S.

See SLP2 for details

## Chomsky Normal Form (CNF)

| $N$ | Finite set of non-terminal symbols | NP, VP, S |
| :--- | :---: | :--- |
| $\Sigma$ | Finite alphabet of terminal symbols | the, dog, eat |
| $R$ | Set of production rules, each of the form <br> $A \rightarrow \beta, \beta \in(\Sigma \cup N) *$ <br> where $\beta=$ a single terminal in $\Sigma$ or <br> two non-terminals in $N$ | $\mathrm{S} \rightarrow \mathrm{NP}$ VP <br> Noun $\rightarrow$ dog |
| $S$ | A designated start symbol |  |

## Chomsky Normal Form (CNF)

Any CFG can be converted into a weakly equivalent CNF (recognizing the same set of sentences as existing in the grammar but differing in their derivation).

## Case 1: mix of terminals and non-terminals

Case 2: more than 2
non-terminals

INF-VP $\rightarrow$ to VP

INF-VP $\rightarrow$ TO VP
$\mathrm{TO} \rightarrow$ to


## CNF Conversion

Case 3: single nonterminal
$\mathrm{A} \rightarrow{ }^{*} \mathrm{~B}$
$B \rightarrow \gamma$
$A \rightarrow \gamma$

| $S$ | $\rightarrow$ NP VP |
| ---: | :--- |
| VP | $\rightarrow$ VBD NP |
| VP | $\rightarrow$ VP PP |
| Nominal | $\rightarrow$ Nominal PP |
| Nominal | $\rightarrow$ NN |
| Nominal | $\rightarrow$ NNS |
| Nominal | $\rightarrow$ PRP |
| PP | $\rightarrow$ IN NP |
| NP | $\rightarrow$ DT NN |
| NP | $\rightarrow$ Nominal |
| NP | $\rightarrow$ PRP\$ Nominal |


| VBD | $\rightarrow$ shot |
| ---: | :--- |
| DT | $\rightarrow$ an $/$ my |
| NN | $\rightarrow$ elephant |
| NNS | $\rightarrow$ pajamas |
| PRP | $\rightarrow$ I |
| PRPS | $\rightarrow$ my |
| IN | $\rightarrow$ in |

## I shot an elephant in my pajamas

## CNF Conversion

## Case 3: single nonterminal

$$
\begin{aligned}
& \mathrm{A} \rightarrow{ }^{*} \mathrm{~B} \\
& \mathrm{~B} \rightarrow \gamma
\end{aligned}
$$

$A \rightarrow \gamma$

| $\mathrm{S} \rightarrow \mathrm{NPVP}$ |  |
| :---: | :---: |
| $\mathrm{VP} \rightarrow \mathrm{VBD}$ NP |  |
| $\mathrm{VP} \rightarrow \mathrm{VPPP}$ | VBD $\rightarrow$ shot |
| Nominal $\rightarrow$ Nominal PP | DT $\rightarrow$ an $\mid \mathrm{my}$ |
| $\text { Nominal } \rightarrow \begin{gathered} \text { pajamas } \\ \text { elephant }\|\mid \end{gathered}$ | PRP $\rightarrow$ I |
| PP $\rightarrow$ IN NP | PRP\$ $\rightarrow$ my |
| NP $\rightarrow$ DT NN | $\mathrm{IN} \rightarrow$ in |
| $N P \rightarrow \begin{gathered} \text { pajamas } \\ \text { elephant }\|\mid \end{gathered}$ |  |
| NP $\rightarrow$ PRP\$ Nominal |  |

## I shot an elephant in my pajamas

## CYK Parsing

For parsing from a grammar expressed in CNF
Bottom-up dynamic programming

function CKY-PARSE(words, grammar) returns table

$$
\begin{aligned}
& \text { for } j \leftarrow \text { from } 1 \text { to } \operatorname{LENGTH}(\text { words }) \text { do } \\
& \text { for all }\{A \mid A \rightarrow \text { words }[j] \in \operatorname{grammar}\} \\
& \quad \text { table }[j-1, j] \leftarrow \text { table }[j-1, j] \cup A \\
& \text { for } i \leftarrow \text { from } j-2 \text { downto } 0 \text { do } \\
& \quad \text { for } k \leftarrow i+1 \text { to } j-1 \text { do } \\
& \quad \text { for all }\{A \mid A \rightarrow B C \in \text { grammar and } B \in \text { table }[i, k] \text { and } C \in \text { table }[k, j]\} \\
& \quad \text { table }[i, j] \leftarrow \text { table }[i, j] \cup A
\end{aligned}
$$

Figure 13.5 The CKY algorithm.

| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| NP, PRP <br> $[0,1]$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| NP, PRP <br> $[0,1]$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | VBD <br> $[1,2]$ |  |  |  |  |  |
|  |  | DT <br> $[2,3]$ |  |  |  |  |
|  |  | NP, NN <br> $[3,4]$ |  |  |  |  |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NP, PRP <br> $[0,1]$ |  |  |  |  |  |  |
| $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ |  |  |  |  |  |  |
| $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ |  |  |  |  |  |  |
| $\begin{gathered} \text { NP, NN } \\ {[3,4]} \end{gathered}$ |  |  |  |  |  |  |
|  |  |  |  | $\underset{[4,5]}{\mathrm{IN}}$ |  |  |
| What phrases can be formed from "I shot an elephant in my pajamas" |  |  |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ |  |
|  |  |  |  |  |  | $\begin{aligned} & \text { NNS } \\ & {[6,7]} \end{aligned}$ |

In CNF, each non-terminal generates two non-terminals

$$
\begin{aligned}
& S \rightarrow \text { NP VP } \\
& {\left[_ { S } \left[\left[_{N P} I\right]\left[{ }_{V P} \text { shot an elephant in my pajamas }\right]\right.\right. \text { ] }}
\end{aligned}
$$

If the left-side non-terminal spans tokens $i-j$, the right side must also span $i-j$, and there must be a single position $k$ that separates them.

|  | 1 | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s \rightarrow$ npvp | NP, PRP $[0,1]$ |  |  |  |  |  |  |
| $\mathrm{vp} \rightarrow \mathrm{vBENP}$ | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| PP $\rightarrow$ NNP | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ |  |  |  |  |  |  |
| $\mathrm{nP} \rightarrow$ DTNN |  |  |  |  |  |  |  |
| NP $\rightarrow$ paiamas |  |  |  |  |  |  |  |
| NP $\rightarrow$ PRPS Sominal | $\begin{gathered} \text { NP, NN } \\ {[3,4]} \end{gathered}$ |  |  |  |  |  |  |
| $\underset{\text { vie } \rightarrow \text { shot }}{\substack{\text { vo } \\ \text { Di }}}$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| PRPs $\rightarrow$ my |  |  |  |  |  |  |  |
| N $\rightarrow$ in |  |  |  |  | $[4,5]$ |  |  |
|  | Does any rule generate PRP VBD? |  |  |  |  | $\begin{gathered} \text { PRP\$ } \\ {[5,6]} \end{gathered}$ |  |
|  |  |  |  |  |  |  | $\begin{aligned} & \text { NNS } \\ & {[6,7]} \end{aligned}$ |


|  | I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { NP, PRP } \\ {[0,1]} \end{gathered}$ | $\varnothing$ |  |  |  |  |  |
|  |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ |  |  |  |  |  |
|  |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ |  |  |  |  |
|  |  |  |  | $\begin{aligned} & \text { NP, NN } \\ & {[3,4]} \end{aligned}$ |  |  |  |
| $\begin{aligned} \mathrm{PRP} & \rightarrow \mathrm{I} \\ \mathrm{PRPS} & \rightarrow \mathrm{my} \\ \mathrm{IN} & \rightarrow \text { in } \end{aligned}$ |  |  |  |  | $\underset{[4,5]}{\mathbb{N}}$ |  |  |
|  | Does | rule ge |  |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ |  |
|  |  |  |  |  |  |  | NNS $[6,7]$ |


|  | I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { NP, PRP } \\ {[0,1]} \end{gathered}$ | $\varnothing$ |  |  |  |  |  |
|  |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ |  |  |  |  |  |
|  |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ |  |  |  |  |
|  |  |  |  | $\begin{aligned} & \text { NP, NN } \\ & {[3,4]} \end{aligned}$ |  |  |  |
| $\begin{aligned} \mathrm{PRP} & \rightarrow \mathrm{I} \\ \mathrm{PRPS} & \rightarrow \mathrm{my} \\ \mathrm{IN} & \rightarrow \text { in } \end{aligned}$ |  |  |  |  | $\underset{[4,5]}{\mathbb{N}}$ |  |  |
|  | Does | rule ge |  |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ |  |
|  |  |  |  |  |  |  | NNS $[6,7]$ |


|  | 1 | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ | $\varnothing$ |  |  |  |  |
| $\begin{aligned} \text { Nominal } & \left.\rightarrow \begin{array}{c} \text { pajamas } \\ \text { elephant } \end{array} \right\rvert\, \\ \mathrm{PP} & \rightarrow \text { IN NP } \\ \mathrm{NP} & \rightarrow \text { DT NN } \end{aligned}$ |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ |  |  |  |  |
|  |  |  |  | NP, NN [3,4] |  |  |  |
|  |  |  |  |  | $\underset{[4,5]}{\mathbb{N}}$ |  |  |
|  | Two pos | places olit k | or that |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ |  |
|  |  |  |  |  |  |  | NNS <br> [6,7] |


|  | I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{NP}, \mathrm{PRP}$ <br> $[0,1]$   VBD <br> $[1,2]$ |  |  |  |  |  |  |
| S $\rightarrow$ NP VP <br> VP $\rightarrow \mathrm{VBD} \mathrm{NP}$ <br> VP $\rightarrow$ VP PP <br> Nominal $\rightarrow$ Nominal PP |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  | $\begin{aligned} & \text { NP, NN } \\ & {[3,4]} \end{aligned}$ |  |  |  |
|  |  |  |  |  | $\underset{[4,5]}{\mathbb{N}}$ |  |  |
|  | Two possible | places plit k | or that |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ |  |
|  |  |  |  |  |  |  | NNS $[6,7]$ |


|  | I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { NP, PRP } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |  |  |  |
|  |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ | $\varnothing$ |  |  |  |  |
|  |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { NP } \rightarrow \text { PRPS Nominial } \\ \text { VBD } \rightarrow \text { shol } \\ \text { DT } \rightarrow \text { an Imy } \end{gathered}$ |  |  |  | NP, NN $[3,4]$ |  |  |  |
| $\begin{aligned} & \text { PRP } \rightarrow 1 \\ & \text { PRPS } \rightarrow \text { my } \\ & \text { N } \rightarrow \text { in } \end{aligned}$ |  |  |  |  | $\begin{gathered} \mathrm{IN} \\ {[4,5]} \end{gathered}$ |  |  |
|  | Does | rule g NN? |  |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ |  |
|  |  |  |  |  |  |  | $\begin{aligned} & \text { NNS } \\ & {[6,7]} \end{aligned}$ |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NP, PRP <br> $[0,1]$ $\varnothing$ $\varnothing$    VBD <br> $[1,2]$ | $\varnothing$ |  |  |  |  |  |


| 1 | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NP, PRP [0,1] | $\varnothing$ |  |  |  |  |  |
|  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ |  |  |  |  |  |
|  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \mathrm{NP} \\ {[2,4]} \end{gathered}$ |  |  |  |
|  |  |  | $\begin{aligned} & \mathrm{NP}, \mathrm{NN} \\ & {[3,4]} \end{aligned}$ |  |  |  |
|  |  |  |  | $\underset{[4,5]}{\mathbb{N}}$ |  |  |
| Two possible places look for that split k |  |  |  |  | $\begin{gathered} \text { PRP\$ } \\ {[5,6]} \end{gathered}$ |  |
|  |  |  |  |  |  | NNS <br> [6,7] |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NP, PRP <br> $[0,1]$ $\varnothing$ $\varnothing$    |  |  |  |  |  |  |
| VBD <br> $[1,2]$ | $\varnothing$ | VP <br> $[1,4]$ |  |  |  |  |





| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


|  | $\begin{gathered} \text { NP, PRP } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} S \\ {[0,4]} \end{gathered}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ | $\varnothing$ | $\begin{gathered} \text { VP } \\ {[1,4]} \end{gathered}$ |  |  |  |
|  |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \mathrm{NP} \\ {[2,4]} \end{gathered}$ |  |  |  |
| $\begin{gathered} \text { NP } \rightarrow \text { Prps Sominial } \\ \text { VsD } \rightarrow \text { shot } \\ \text { DT } \rightarrow \text { an Imy } \end{gathered}$ |  |  |  | $\begin{gathered} \text { NP, NN } \\ {[3,4]} \end{gathered}$ |  |  |  |
| $\begin{aligned} \text { PRP } & \rightarrow \text { 1 } \\ \text { PRPS } & \rightarrow \text { my } \\ \text { IN } & \rightarrow \text { in } \end{aligned}$ |  |  |  |  | $\begin{gathered} \mathrm{IN} \\ {[4,5]} \end{gathered}$ |  |  |
|  |  |  |  |  |  | PRP\$ <br> [5,6] |  |
|  |  |  |  |  |  |  | $\begin{aligned} & \text { NNS } \\ & {[6,7]} \end{aligned}$ |



| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


|  | NP, PRP <br> [0,1] | $\varnothing$ | $\varnothing$ | $\begin{gathered} S \\ {[0,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ | $\varnothing$ | $\begin{gathered} \text { VP } \\ {[1,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |
|  |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \mathrm{NP} \\ {[2,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { NP } \\ {[3,7]} \end{gathered}$ |
| $\begin{gathered} \text { NP } \rightarrow \text { PrPs Sominial } \\ \text { Vso } \rightarrow \text { shol } \\ \text { DT } \rightarrow \text { an Imy } \end{gathered}$ |  |  |  | $\begin{gathered} \text { NP, NN } \\ {[3,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{NP} \\ {[3,7]} \end{gathered}$ |
| $\begin{gathered} \text { PRP } \rightarrow 1 \\ \text { PRPS } \rightarrow \text { my } \\ \text { N } \rightarrow \text { in } \end{gathered}$ |  |  |  |  | $\begin{gathered} \mathrm{IN} \\ {[4,5]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \text { PP } \\ {[4,7]} \end{gathered}$ |
|  |  |  |  |  |  | $\begin{gathered} \text { PRP\$ } \\ {[5,6]} \end{gathered}$ | $\begin{gathered} \text { NP } \\ {[5,7]} \end{gathered}$ |
|  |  |  |  |  |  |  | $\begin{aligned} & \text { NNS } \\ & {[6,7]} \end{aligned}$ |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


|  | $\begin{gathered} \text { NP, PRP } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\begin{array}{c\|c}  & S \\ \hline & \\ {[0,4]} \end{array}$ |  | $\varnothing$ |  | $\begin{gathered} \mathrm{NP} \\ {[3,7]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} \mathrm{S} & \rightarrow \mathrm{NP} \mathrm{VP} \\ \mathrm{VP} & \rightarrow \mathrm{VBD} \mathrm{NP} \\ \mathrm{VP} & \rightarrow \mathrm{VPPP} \end{aligned}$ |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ | $\varnothing$ | $\begin{gathered} \text { VP } \\ {[1,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |
|  |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \text { NP } \\ {[2,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |
|  |  |  |  | $\begin{gathered} \text { NP, NN } \\ {[3,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{NP} \\ {[3,7]} \end{gathered}$ |
| $\begin{aligned} & \text { ot } \rightarrow \text { an } \mid m y \\ & \text { PRP } \rightarrow 1 \\ & \text { PRPs } \rightarrow \text { my } \\ & \text { iN } \rightarrow \text { in } \end{aligned}$ |  |  |  |  | $\begin{gathered} \mathrm{IN} \\ {[4,5]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \mathrm{PP} \\ {[4,7]} \end{gathered}$ |
|  |  |  |  |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ | $\begin{gathered} \text { NP } \\ {[5,7]} \end{gathered}$ |
|  |  |  |  |  |  |  | NNS <br> [6,7] |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |



| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |



| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |



| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |



| I | shot | an | elephant | in | my | pajamas |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| । | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


|  | $\begin{gathered} \text { NP, PRP } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{S} \\ {[0,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S$ $\rightarrow$ NP VP <br> VP $\rightarrow \mathrm{VBD} \mathrm{NP}$ <br> VP $\rightarrow \mathrm{VPPP}$ <br> Nominal $\rightarrow$ Nominal PP |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ | $\varnothing$ | $\begin{gathered} \text { VP } \\ {[1,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{VP}_{1}, \mathrm{VP}_{2} \\ {[1,7]} \end{gathered}$ |
|  |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \mathrm{NP} \\ {[2,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{NP} \\ {[2,7]} \end{gathered}$ |
|  |  |  |  | NP, NN $[3,4]$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{NP} \\ {[3,7]} \end{gathered}$ |
|  |  |  |  |  | $\begin{gathered} \mathrm{IN} \\ {[4,5]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \text { PP } \\ {[4,7]} \end{gathered}$ |
|  |  |  |  |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ | $\begin{gathered} \mathrm{NP} \\ {[5,7]} \end{gathered}$ |
|  |  |  |  |  |  |  | $\begin{aligned} & \text { NNS } \\ & {[6,7]} \end{aligned}$ |



| । | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


|  | $\begin{gathered} \text { NP, PRP } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} S \\ {[0,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{aligned} & \mathrm{S}_{1}, \mathrm{~S}_{2} \\ & {[0,7]} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S$ $\rightarrow$ NP VP <br> VP $\rightarrow$ VBD NP <br> VP $\rightarrow$ VP PP <br> Nominal $\rightarrow$ Nominal PP |  | $\begin{aligned} & \text { VBD } \\ & {[1,2]} \end{aligned}$ | $\varnothing$ | $\begin{gathered} \text { VP } \\ {[1,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{VP}_{1}, \mathrm{VP}_{2} \\ {[1,7]} \end{gathered}$ |
| Nominal $\rightarrow$pajamas \| <br> elephant $\|\mid$ <br> PP $\rightarrow \mathbb{I N N P}$ <br> NP $\rightarrow$ DT NN |  |  | $\begin{gathered} \text { DT } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \mathrm{NP} \\ {[2,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{NP} \\ {[2,7]} \end{gathered}$ |
|  |  |  |  | $\begin{gathered} \mathrm{NP}, \mathrm{NN} \\ {[3,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \mathrm{NP} \\ {[3,7]} \end{gathered}$ |
|  |  |  |  |  | $\underset{\substack{\mathrm{IN} \\[4,5]}}{ }$ | $\varnothing$ | $\begin{gathered} \text { PP } \\ {[4,7]} \end{gathered}$ |
|  | Success! We've recognized a total of two valid parses |  |  |  |  | $\begin{aligned} & \text { PRP\$ } \\ & {[5,6]} \end{aligned}$ | $\begin{gathered} \mathrm{NP} \\ {[5,7]} \end{gathered}$ |
|  |  |  |  | Complexity? |  |  | $\begin{aligned} & \text { NNS } \\ & {[6,7]} \end{aligned}$ |

## CFG

CYK allows us to:

- check whether a sentence in grammatical in the language defined by the CFG
- enumerate all possible parses for a sentence CFG

But it doesn't tell us on which one of those possible parses is most likely

- might help to to disambiguate
-> Probabilistic context-free grammar


## Probabilistic Context-free Grammar (PCFG)

Probabilistic context-free grammar: each production is also associated with a probability.

| $N$ | Finite set of non-terminal symbols | $\mathrm{NP}, \mathrm{VP}, \mathrm{S}$ |
| :--- | :---: | :--- |
| $\Sigma$ | Finite alphabet of terminal symbols | the, dog, eat |
| $R$ | Set of production rules, each of the form <br> $A \rightarrow \beta[p], \beta \in(\Sigma \cup N) *$ <br> $p=P(\beta \mid A)$ | $\mathrm{S} \rightarrow \mathrm{NP}$ VP <br> Noun $\rightarrow$ dog |
| $S$ | A designated start symbol |  |

## Probabilistic Context-free Grammar (PCFG)

We can then calculate the probability of a parse for a given sentence

For a given parse tree $T$ for sentence $S$ comprised of $n$ rules from $R$ (each $A \rightarrow \beta$ ):

$$
P(T)=\prod_{i=1}^{n} P(\beta \mid A)
$$

In practice, we often want to find the single best parse with the highest probability for a given tree $S$ :

$$
\begin{aligned}
& T^{*}(S)=\operatorname{argmax}_{T} P(T \mid S)=\operatorname{argmax}_{T} \frac{P(S \mid T) P(T)}{P(S)} \\
& =\operatorname{argmax}_{T} P(S \mid T) P(T)=\operatorname{argmax}_{T} P(T) \\
& P(S \mid T)=1, \text { since } T \text { includes all the words of } S
\end{aligned}
$$

We calculate the max probability parse using CKY by storing the max probability of each phrase within each cell as we build it up.

## Probabilistic CYK for PCFG

function Probabilistic-CKY(words,grammar) returns most probable parse and its probability
for $j \leftarrow$ from 1 to $\operatorname{LENGTH}$ (words) do
for all $\{A \mid A \rightarrow$ words $[j] \in$ grammar $\}$

$$
\text { table }[j-1, j, A] \leftarrow P(A \rightarrow \text { words }[j])
$$

for $i \leftarrow$ from $j-2$ downto 0 do
for $k \leftarrow i+1$ to $j-1$ do
for all $\{A \mid A \rightarrow B C \in$ grammar, and table $[i, k, B]>0$ and table $[k, j, C]>0\}$
if $($ table $[i, j, A]<P(A \rightarrow B C) \times$ table $[i, k, B] \times \operatorname{table}[k, j, C])$ then
table $[i, j, A] \leftarrow P(A \rightarrow B C) \times$ table $[i, k, B] \times$ table $[k, j, C]$ $\operatorname{back}[i, j, A] \leftarrow\{k, B, C\}$
return BUILD_TREE(back[1, LENGTH(words), $S]$ ), table[1, LENGTH(words), $S$ ]

## Estimate The Probabilities

Using the treebank to count the statistics

$$
P(\beta \mid A)=\frac{\operatorname{Count}(A \rightarrow \beta)}{\sum_{\gamma} \operatorname{Count}(A \rightarrow \gamma)}=\frac{\operatorname{Count}(A \rightarrow \beta)}{\operatorname{Count}(A)}
$$

We can also estimate the probabilities using a (non-probabilistic) parser

- Parse the corpus, compute the statistics, and normalize the probabilities
- Might need to use the inside-outside algorithm for ambiguous sentences (see SLP2,3)

| A |  | $\beta$ | $P(\beta \mid N P)$ |
| :---: | :---: | :---: | :---: |
| NP | $\rightarrow$ | NP PP | 0.092 |
| NP | $\rightarrow$ | DT NN | 0.087 |
| NP | $\rightarrow$ | NN | 0.047 |
| NP | $\rightarrow$ | NNS | 0.042 |
| NP | $\rightarrow$ | DT JJ NN | 0.035 |
| NP | $\rightarrow$ | NNP | 0.034 |
| NP | $\rightarrow$ | NNP NNP | 0.029 |
| NP | $\rightarrow$ | JJ NNS | 0.027 |
| NP | $\rightarrow$ | QP -NONE- | 0.018 |
| NP | $\rightarrow$ | NP SBAR | 0.017 |
| NP | $\rightarrow$ | NP PP-LOC | 0.017 |
| NP | $\rightarrow$ | JJ NN | 0.015 |
| NP | $\rightarrow$ | DT NNS | 0.014 |
| NP | $\rightarrow$ | CD | 0.014 |
| NP | $\rightarrow$ | NN NNS | 0.013 |
| NP | $\rightarrow$ | DT NN NN | 0.013 |
| NP | $\rightarrow$ | NP CC NP | 0.013 |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { PRP:0.04 } \\ {[0,1]} \end{gathered}$ |  |  |  |  |  |  |
| $\begin{gathered} \text { VBD:0.04 } \\ {[1,2]} \end{gathered}$ |  |  |  |  |  |  |
| $\begin{gathered} \text { DT:0.05 } \\ {[2,3]} \end{gathered}$ |  |  |  |  |  |  |
|  |  |  | $\begin{gathered} \mathrm{NN}: 0.03 \\ {[3,4]} \end{gathered}$ |  |  |  |
| Probaiblity of a terminal (word) given its tag |  |  |  | $\begin{aligned} & \text { IN:0.10 } \\ & {[4,5]} \end{aligned}$ |  |  |
|  |  |  |  |  | PRP\$: |  |
| $P(A \rightarrow \beta)$ |  |  |  |  | $\begin{aligned} & 0.12 \\ & 15 \mathrm{kl} \end{aligned}$ |  |
|  |  |  |  |  |  | $\begin{gathered} \text { NNS:0.01 } \\ {[6,7]} \end{gathered}$ |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { PRP:0.04 } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |  |  |  |
|  | $\begin{gathered} \text { VBD:0.04 } \\ {[1,2]} \end{gathered}$ | $\varnothing$ |  |  |  |  |
|  |  | $\begin{gathered} \text { DT:0.05 } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \mathrm{NP:} \\ 0.00015 \\ {[2,4]} \end{gathered}$ |  |  |  |
|  |  |  | $\begin{gathered} \text { NN:0.03 } \\ {[3,4]} \end{gathered}$ |  |  |  |
|  |  |  |  | $\begin{gathered} \text { IN:0.10 } \\ {[4,5]} \end{gathered}$ |  |  |
|  |  |  |  |  | $\begin{gathered} \text { PRP\$:0.12 } \\ {[5,6]} \end{gathered}$ |  |
| $b l e(2,4, N P)=P(\mathrm{NP} \rightarrow \mathrm{DT} \mathrm{NN}) \times \operatorname{table}(2,3, D T) \times \operatorname{table}(3,4, N N)$ |  |  |  |  |  | $\begin{gathered} \text { NNS:0.01 } \\ {[6,7]} \end{gathered}$ |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| $\begin{gathered} \text { PRP:0.04 } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { VBD:0.04 } \\ {[1,2]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \text { VP: } \\ 0.0000006 \\ {[1,4]} \end{gathered}$ |  |  |  |
|  |  | $\begin{gathered} \text { DT:0.05 } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \text { NP: } \\ 0.00015 \end{gathered}$ $[2,4]$ |  |  |  |
|  |  |  | $\begin{gathered} \text { NN:0.03 } \\ {[3,4]} \end{gathered}$ |  |  |  |
|  |  |  |  | $\begin{gathered} \text { IN:0.10 } \\ {[4,5]} \end{gathered}$ |  |  |
| We just calculated this value and can use it now |  |  |  |  | $\begin{gathered} \text { PRP\$:0.12 } \\ {[5,6]} \end{gathered}$ |  |
| $(1,4, V P)$ | $P(\mathrm{VP} \rightarrow$ | D NP) $\times$ | able(1,2, V | D) $\times t a$ | 2, 4, NP) | $\begin{gathered} \text { NNS:0.01 } \\ {[6,7]} \end{gathered}$ |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { PRP: -3.21 } \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { S: -19.2 } \\ {[0,4]} \end{gathered}$ |  |  |  |
|  | $\begin{gathered} \text { VBD: -3.21 } \\ {[1,2]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \text { VP: }-14.3 \\ {[1,4]} \end{gathered}$ |  |  |  |
|  |  | $\begin{gathered} \text { DT: -3.0 } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \text { NP: -8.8 } \\ {[2,4]} \end{gathered}$ |  |  |  |
|  |  |  | NN: -3.5 $[3,4]$ |  |  |  |
|  |  |  |  | $\begin{gathered} \text { IN: - } 2.3 \\ {[4,5]} \end{gathered}$ |  |  |
| Note these values are getting very small! Better to add in log space |  |  |  |  | $\begin{aligned} & \text { PRP\$: } \\ & -2.12 \\ & {[5,6]} \end{aligned}$ |  |
|  |  |  |  |  |  | NNS: -4.6 $[6,7]$ |


| I shot an elephant in mv pajamasPRP: -3.21 <br> $[0,1]$ $\varnothing$ $\varnothing$ $\mathrm{S}:-19.2$ <br> $[0,4]$ $\varnothing$ | $\varnothing$ |
| :---: | :---: |


| 1 | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PRP: -3.21 <br> $[0,1]$ $\varnothing$ $\varnothing$ $\mathrm{S}:-19.2$ <br> $[0,4]$ $\varnothing$ $\varnothing$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| $\begin{gathered} \text { VBD: -3.21 } \\ {[1,2]} \end{gathered}$ |  | $\varnothing$ | $\begin{gathered} \text { VP: -14.3 } \\ {[1,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { VP: -30.2 } \\ {[1,7]} \end{gathered}$ |
|  |  | $\begin{gathered} \text { DT: -3.0 } \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \text { NP: -8.8 } \\ {[2,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { NP: - } 24.7 \\ {[2,7]} \end{gathered}$ |
|  |  |  | NN: -3.5 $[3,4]$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { NP: -19.4 } \\ {[3,7]} \end{gathered}$ |
|  |  |  |  | $\begin{gathered} \text { IN: -2.3 } \\ {[4,5]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \text { PP: -13.6 } \\ {[4,7]} \end{gathered}$ |
| For any phrase type spanning [i,j], we only need to keep the max probability given the assumptions of a PCFG |  |  |  |  | $\begin{aligned} & \text { PRP\$: } \\ & -2.12 \\ & {[5,6]} \end{aligned}$ | $\begin{gathered} \text { NP: -9.0 } \\ {[5,7]} \end{gathered}$ |
|  |  |  |  |  |  | $\begin{gathered} \text { NNS: -4.6 } \\ {[6,7]} \end{gathered}$ |


| I | shot | an | elephant | in | my | pajamas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { PRP: }-3.21 \\ {[0,1]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { S: -19.2 } \\ {[0,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { S: -35.7 } \\ {[0,7]} \end{gathered}$ |
|  | $\begin{gathered} \text { VBD: }-3.21 \\ {[1,2]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \text { VP: - } 14.3 \\ {[1,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { VP:- }-30.2 \\ {[1,7]} \end{gathered}$ |
|  |  | $\begin{gathered} \text { DT: }-3.0 \\ {[2,3]} \end{gathered}$ | $\begin{gathered} \text { NP: -8.8 } \\ {[2,4]} \end{gathered}$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { NP: -24.7 } \\ {[2,7]} \end{gathered}$ |
|  |  |  | NN: -3.5 $[3,4]$ | $\varnothing$ | $\varnothing$ | $\begin{gathered} \text { NP: -19.4 } \\ {[3,7]} \end{gathered}$ |
|  |  |  |  | $\begin{gathered} \text { IN: - } 2.3 \\ {[4,5]} \end{gathered}$ | $\varnothing$ | $\begin{gathered} \text { PP: }-13.6 \\ {[4,7]} \end{gathered}$ |
| For any phrase type spanning [i,j], we only need to keep the max probability given the assumptions of a PCFG |  |  |  |  | $\begin{aligned} & \text { PRP\$: } \\ & -2.12 \\ & {[5,61} \end{aligned}$ | $\begin{gathered} \text { NP: -9.0 } \\ {[5,7]} \end{gathered}$ |
|  |  |  |  |  |  | NNS: -4.6 $[6,7]$ |



## Problems with PCFG

$P(T)=\prod_{i=1}^{n} P(\beta \mid A)$

## Strong independence assumptions:

- Each production (e.g., NP $\rightarrow$ DT NN) is independent of the rest of tree.
- In real use, productions are strongly dependent on their place in the tree.

|  | NP $\rightarrow$ PRP | NP $\rightarrow$ DT NN |
| :---: | :---: | :---: |
|  | Pronoun | Non-Pronoun |
| Subject | $91 \%$ | $9 \%$ |
| Object | $34 \%$ | $66 \%$ |

## Problems with PCFG

$P(T)=\prod_{i=1}^{n} P(\beta \mid A)$
Strong independence assumptions:

|  | NP $\rightarrow$ PRP | NP $\rightarrow$ DT NN |
| :---: | :---: | :---: |
|  | Pronoun | Non-Pronoun |
| Subject | $91 \%$ | $9 \%$ |
| Object | $34 \%$ | $66 \%$ |

- With maximum likelihood estimator on Swithboard dataset:

$$
\begin{aligned}
& P(N P \rightarrow D T N N)=0.28 \\
& P(N P \rightarrow P R P)=0.25
\end{aligned}
$$

## Splitting Non-Terminals/ Parent Annotation

Rather than having a single rule for each non-terminal $P(N P \rightarrow$ DT NN), we can condition on some context (Johnson 1998)

- $P_{\text {subject }}$ (NP $\rightarrow$ DT NN)
- $P_{\text {object }}(N P \rightarrow D T N N)$

More generally, we can encode context by annotating each node in a tree with its parent (parent annotation)

- This lets us to learn different probabilities for:
- $N P^{S}$ (subject)
- $\mathrm{NP}_{\mathrm{Vp}}$ (object)


This Dramatically increases the size of the grammar $\rightarrow$ less traıning data for each production (data sparsity)

Modern approaches search for best splits that maximize the training data likelihood (Petrov et al 2006)

## Problems with PCFGs

## Lack of lexical dependency: Lexical information in a PCFG has little influence on the overall parse structure

- The identity of the verbs, nouns, and prepositions might be crucial to disambiguate the parses.


Figure 14.5 Two possible parse trees for a prepositional phrase attachment ambiguity. The left parse is the sensible one, in which "into a bin" describes the resulting location of the sacks. In the right incorrect parse, the sacks to be dumped are the ones which are already "into a bin", whatever that might mean.


Figure 14.7 An instance of coordination ambiguity. Although the left structure is intuitively the correct one, a PCFG will assign them identical probabilities since both structures use exactly the same set of rules. After Collins (1999).

## Lexicalized PCFG

## Annotate each node with its head + POS tag of head



Figure 14.10 A lexicalized tree, including head tags, for a WSJ sentence, adapted from Collins (1999). Below we show the PCFG rules needed for this parse tree, internal rules on the left, and lexical rules on the right.

## Lexicalized PCFG

Annotate each node with its head + POS tag of head

We can't estimate probabilities for such fine-grained productions well:

$$
\frac{\operatorname{Count}(V P(\text { dumped }, V B D) \rightarrow V B D(\text { dumped, } V B D) N P(\text { sacks, } N N S) P P(\text { into }, P))}{\operatorname{Count}(V P(\text { dumped }, V B D))}
$$

Different models make different independent assumptions to make this quantity tractable (Collins 1999, Charniak 1997)

## Parameters in a Lexicalized PCFG

An example parameter in a PCFG:

$$
p(S \rightarrow N P \vee P)
$$

An example parameter in a Lexicalized PCFG:

$$
p\left(S(\text { saw }) \rightarrow_{2} N P(\text { man }) V P(s a w)\right.
$$

| $N$ | Finite set of non-terminal symbols | $\mathrm{NP}, \mathrm{VP}, \mathrm{S}$ |
| :--- | :---: | :--- |
| $\Sigma$ | Finite alphabet of terminal symbols | the, dog, eat |
| $R$ | Set of production rules |  |
| $S$ | A designated start symbol |  |

R is a set of rules which take one of three forms:

$$
\begin{aligned}
& \circ X(h) \rightarrow_{1} Y_{1}(h) Y_{2}(w) \text { for } X \in N \text {, and } Y_{1}, Y_{2} \in N \text {, and } h, w \in \sum \\
& \circ X(h) \rightarrow_{2} Y_{1}(w) Y_{2}(h) \text { for } X \in N \text {, and } Y_{1}, Y_{2} \in N \text {, and } h, w \in \sum \\
& \circ X(h) \rightarrow h \text { for } X \in N \text {, and } h \in \sum
\end{aligned}
$$

## Parsing with Lexicalized PCFG

For PCFG in Chomsky Normal Form, we can parse an $n$ word sentence in

$$
O\left(n^{3} \times|N|^{3}\right)
$$

Lexicalized PCFG: the grammar looks just like a Chomsky normal form CFG, but with potentially $O\left(|\Sigma|^{2} \times|N|^{3}\right)$ possible rules.
Naively, parsing using the dynamic programming algorithm will take $O\left(n^{3} \times\right.$ $\left.|\Sigma|^{2} \times|N|^{3}\right)$ time. But $|\Sigma|^{2}$ can be huge!!
Crucial observation: at most $O\left(n^{2} \times|N|^{3}\right)$ rules can be applicable to a given sentence $w_{1}, w_{2}, \ldots, w_{n}$ of length $n$. This is because any rules which contain a lexical item that is not one of $w_{1}, w_{2}, \ldots, w_{n}$, can be safely discarded.
The result: we can parse in $O\left(n^{5} \times|N|^{3}\right)$ time.

