

Part-of-Speech Tagging and Hidden Markov Model

Bonan Min

bonanmin@gmail.com

Some slides are based on class materials from Thien Huu Nguyen and Ralph Grishman

Parts of Speech (POS)

Role of parts-of-speech in grammar

- 'preterminals'
- Rules stated in terms of classes of words sharing syntactic properties

noun

verb

adjective

...

Parts of Speech (POS)

The distributional hypothesis: Words that appear in similar contexts have similar representations (and similar meanings)

Substitution test for POS: if a word is replaced by another word, does the sentence remain **grammatical**?

*He noticed the **elephant** before anybody else*

dog

cat

point

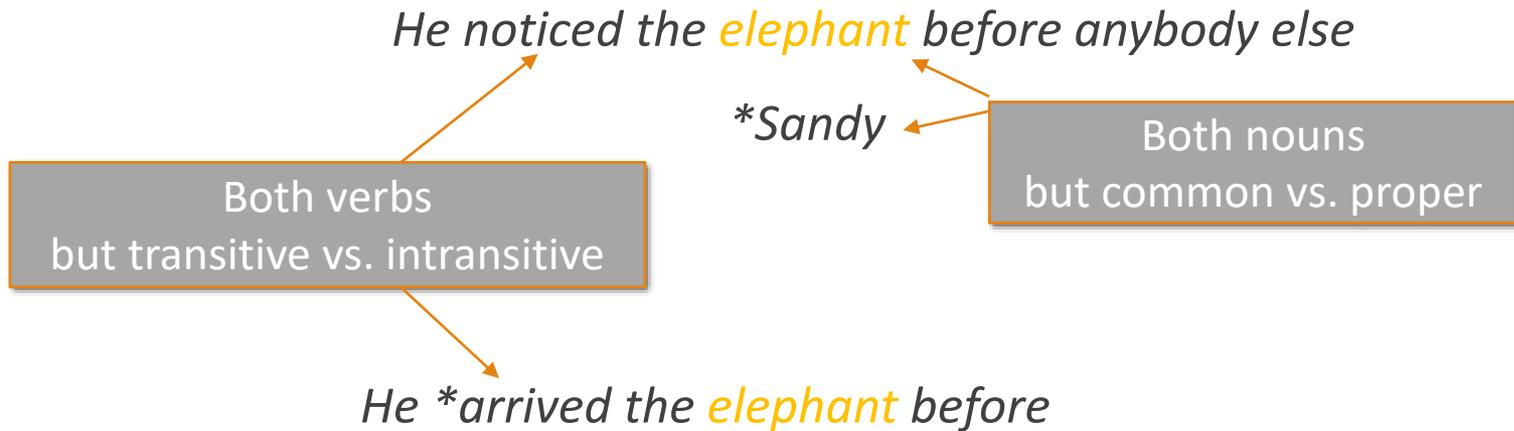
features

**what*

**and*

Substitution Test

These can often be too strict; some contexts admit substitutability for some pairs but not others.



Parts of Speech (POS)

Nouns	People, places, things, actions-made-nouns (“I like swimming ”). Inflected for singular/plural
Verbs	Actions, processes. Inflected for tense, aspect, number, person
Adjectives	Properties, qualities. Usually modify nouns
Adverbs	Qualify the manner of verbs (“She ran downhill extremely quickly yesterday ”)
Determiner	Mark the beginning of a noun phrase (“ a dog”)
Pronouns	Refer to a noun phrase (he, she, it)
Prepositions	Indicate spatial/temporal relationships (on the table)
Conjunctions	Conjoin two phrases, clauses, sentences (and, or)

POS Tag Sets (Categories)

Most influential tag sets were those defined for projects to produce large POS-annotated corpora:

Brown corpus

- 1 million words from variety of genres
- 87 tags

UPenn Tree Bank

- initially 1 million words of Wall Street Journal
- later retagged Brown
- first POS tags, then full parses
- 45 tags (some distinctions captured in parses)

Cross-lingual considerations for POS tags

Penn Treebank POS Tags

Tag	Description	Example	Tag	Description	Example
CC	coordin. conjunction	<i>and, but, or</i>	SYM	symbol	<i>+, %, &</i>
CD	cardinal number	<i>one, two</i>	TO	“to”	<i>to</i>
DT	determiner	<i>a, the</i>	UH	interjection	<i>ah, oops</i>
EX	existential ‘there’	<i>there</i>	VB	verb base form	<i>eat</i>
FW	foreign word	<i>mea culpa</i>	VBD	verb past tense	<i>ate</i>
IN	preposition/sub-conj	<i>of, in, by</i>	VBG	verb gerund	<i>eating</i>
JJ	adjective	<i>yellow</i>	VBN	verb past participle	<i>eaten</i>
JJR	adj., comparative	<i>bigger</i>	VBP	verb non-3sg pres	<i>eat</i>
JJS	adj., superlative	<i>wildest</i>	VBZ	verb 3sg pres	<i>eats</i>
LS	list item marker	<i>1, 2, One</i>	WDT	wh-determiner	<i>which, that</i>
MD	modal	<i>can, should</i>	WP	wh-pronoun	<i>what, who</i>
NN	noun, sing. or mass	<i>llama</i>	WP\$	possessive wh-	<i>whose</i>
NNS	noun, plural	<i>llamas</i>	WRB	wh-adverb	<i>how, where</i>
NNP	proper noun, sing.	<i>IBM</i>	\$	dollar sign	<i>\$</i>
NNPS	proper noun, plural	<i>Carolinas</i>	#	pound sign	<i>#</i>
PDT	predeterminer	<i>all, both</i>	“	left quote	<i>‘ or “</i>
POS	possessive ending	<i>'s</i>	”	right quote	<i>’ or ”</i>
PRP	personal pronoun	<i>I, you, he</i>	(left parenthesis	<i>[, (, {, <</i>
PRPS	possessive pronoun	<i>your, one’s</i>)	right parenthesis	<i>],), }, ></i>
RB	adverb	<i>quickly, never</i>	,	comma	<i>,</i>
RBR	adverb, comparative	<i>faster</i>	.	sentence-final punc	<i>. ! ?</i>
RBS	adverb, superlative	<i>fastest</i>	:	mid-sentence punc	<i>: ; ... --</i>
RP	particle	<i>up, off</i>			

Penn Treebank POS Tags

Verbs

Tag	Description	Examples
VB	base form (found in imperatives, infinitives and subjunctives)	<ul style="list-style-type: none"> Just do it You should do it He wants to do it
VBD	past tense	<ul style="list-style-type: none"> He ate the food
VBG	present participle (Verb forms in the gerund or present participle; generally end in -ing)	<ul style="list-style-type: none"> He was going to the store She is implementing the algorithm
VBN	past participle	<ul style="list-style-type: none"> The apple was eaten He had expected to go
VBP	present (non 3rd-sing)	<ul style="list-style-type: none"> I am the food You are tall We are tall They do the job
VBZ	present (3rd-sing)	<ul style="list-style-type: none"> She is tall He likes ice cream
MD	modal verbs (All verbs that don't take -s ending in third-person singular present)	<ul style="list-style-type: none"> can, could, dare, may, might, must, ought, shall, should, will, would

4057	will/md
2973	would/md
1483	could/md
1233	can/md
1066	may/md
598	should/md
459	might/md
332	must/md
326	wo/md
246	ca/md

Nouns

Tag	Description	Examples
NN	non-proper, singular or mass	the company
NNS	non-proper, plural	the companies
NNP	proper, singular	Carolina
NNPS	proper, plural	Carolinas

RP (Particle)

Used in combination with a verb

- She turned the paper **over**

verb + particle = phrasal verb, often non-compositional

- turn **down**, rule **out**, find **out**, go **on**

774 up/rp
487 out/rp
301 off/rp
209 down/rp
124 in/rp
98 over/rp
81 on/rp
72 back/rp
46 around/rp
25 away/rp

DT and PDT

DT (Articles)

- Articles (**a**, **the**, **every**, **no**)
- Indefinite determiners (**another**, **any**, **some**, **each**)
- **That**, **these**, **this**, **those** when preceding noun
- **All**, **both** when not preceding another determiner or possessive pronoun

65548	the/dt
26970	a/dt
4405	an/dt
3115	this/dt
2117	some/dt
2102	that/dt
1274	all/dt
1085	any/dt
953	no/dt
778	those/dt

PDT (Predeterminer)

- Determiner-like words that precede an article or possessive pronoun
 - **all** his marbles
 - **both** the girls
 - **such** a good time

263	all/pdt
114	such/pdt
84	half/pdt
24	both/pdt
7	quite/pdt
2	many/pdt
1	nary/pdt

PRP and PRP\$

PRP (personal pronoun)

- Personal pronouns (I, me, you, he, him, it, etc.)
- Reflective pronouns (ending in -self): himself, herself
- Nominal possessive pronouns: mine, yours, hers

```
7854 it/prp
4601 he/prp
3260 they/prp
2323 his/prp$
1792 we/prp
1584 i/prp
1001 you/prp
874 them/prp
694 she/prp
438 him/prp
```

PRP\$ (possessive pronouns)

- Adjectival possessive forms: my, their, its, his, her

```
5013 its/prp$
2364 their/prp$
2323 his/prp$
521 our/prp$
430 her/prp$
328 my/prp$
269 your/prp$
```

Adjectives

JJ (Adjectives)

- General adjectives (**happy** person, **new** house)
- Ordinal numbers (**fourth** cat)

JJR (Comparative adjectives)

- Adjectives with a comparative ending **-er** and comparative meaning (**happier** person)
- More and less (when used as adjectives) (**more** mail)

JJS (Superlative adjectives)

- Adjectives with a superlative ending **-est** and superlative meaning (**happiest** person)
- Most and least (when used as adjectives) (**most** mail)

2002	other/jj
1925	new/jj
1563	last/jj
1174	many/jj
1142	such/jj
1058	first/jj
824	major/jj
715	federal/jj
698	next/jj
644	financial/jj
1498	more/jjr
518	higher/j
432	lower/jj
285	less/jjr
158	better/j
136	smaller/j
122	earlier/j
112	greater/j
93	larger/j
75	bigger/j
695	most/jjs
428	least/jjs
315	largest/jjs
299	latest/jjs
209	biggest/jjs
194	best/jjs
76	highest/jjs
63	worst/jjs
31	lowest/jjs
30	greatest/jjs

Adverbs

RB (Adverbs)

- Most words that end in **-ly** (**highly**, **heavily**)
- Degree words (**quite**, **too**, **very**)
- Negative markers (**not**, **n't**, **never**)

RBR (Comparative adverbs)

- Adverbs with a comparative ending **-er** and comparative meaning, e.g., run **faster**
- **More/less**, e.g., **more** expensive

RBS (Superlative adverbs)

- Adverbs with a superlative ending **-est** and superlative meaning, e.g., run **fastest**
- **Most/least**, e.g., **most** expensive

4410	n't/rb
2071	also/rb
1858	not/rb
1109	now/rb
1070	only/rb
1027	as/rb
961	even/rb
839	so/rb
810	about/rb
804	still/rb
1121	more/rbr
516	earlier/rbr
192	less/rbr
88	further/rbr
82	lower/rbr
75	better/rbr
65	higher/rbr
57	longer/rbr
53	later/rbr
34	faster/rbr
549	most/rbs
21	best/rbs
9	least/rbs
8	hardest/rbs
2	most/rbs jjs
1	worst/rbs
1	rbs/nnp
1	highest/rbs
1	earliest/rbs

IN and CC

IN (preposition, subordinating conjunction)

- All prepositions (except to) and subordinating conjunctions
- He jumped **on** the table **because** he was excited

31111 of/in
22967 in/in
11425 for/in
7181 on/in
6684 that/in
6399 at/in
6229 by/in
5940 from/in
5874 with/in
5239 as/in

CC (coordinating conjunction)

- **And, but, not, or**
- Math operators (**plus, minor, less, times**)
- **For** (meaning “because”)
 - he asked to be transferred, **for** he was unhappy

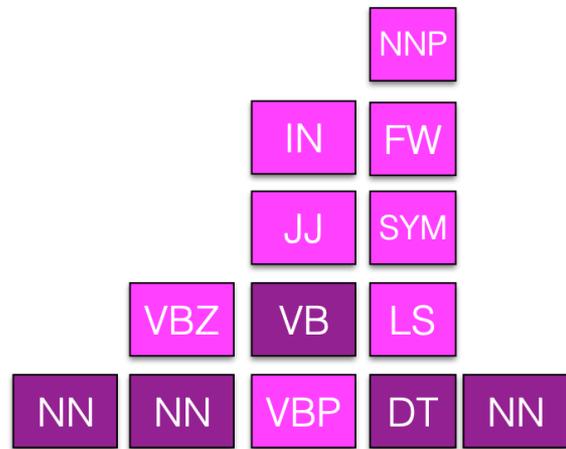
22362 and/cc
4604 but/cc
3436 or/cc
1410 &/cc
94 nor/cc
68 either/cc
53 yet/cc
53 plus/cc
37 both/cc
32 neither/cc

The POS Tagging Task

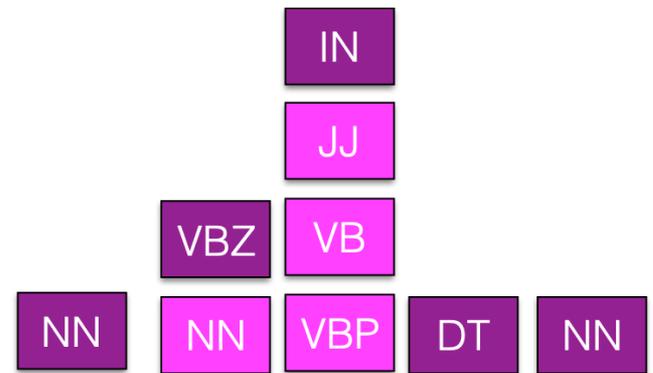
Task: assigning a POS to each word

not trivial: many words have several tags

dictionary only lists possible POS, independent of context



Fruit flies like a banana



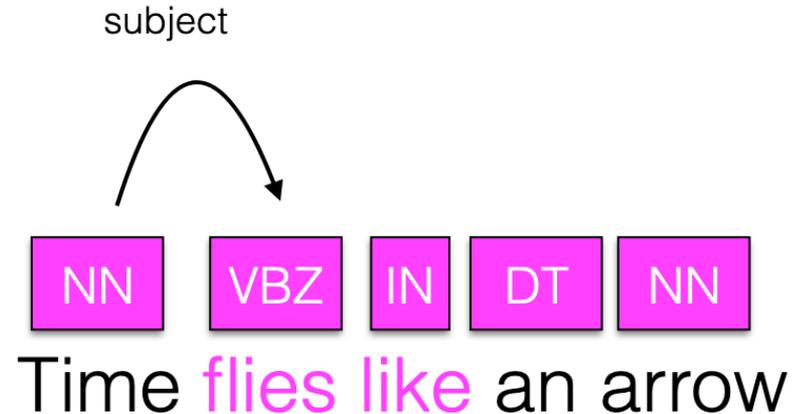
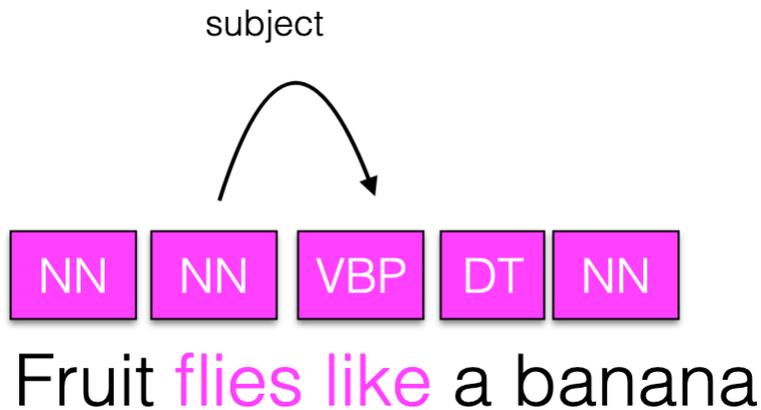
Time flies like an arrow

Why Tag?

POS tagging can help parsing by reducing ambiguity

Can resolve some pronunciation ambiguities for text-to-speech
("desert" – noun: /'dɛzərt/, verb: /dɪ'zɜrt/)

Can resolve some semantic ambiguities



Some Tricky Cases

JJ or VBN

- If it is gradable (can insert “very”) = JJ

- He was very surprised

JJ

- If can be followed by a “by” phrase = VBN. If that conflicts with #1 above, then = JJ

- He was invited by some friends of her

VBN

- He was very surprised by her remarks

JJ

JJ or NNP/NNPS

- Proper names can be adjectives or nouns

- French cuisine is delicious

JJ

- The French tend to be inspired cooks

NNPS

Some Tricky Cases

NN or VBG

- Only nouns can be modified by adjectives; only gerunds(-ing) can be modified by adverbs
- Good **cooking** is something to enjoy
- **Cooking** well is a useful skill

NN

VBG

IN or RP

- If it can precede or follow the noun phrase = RP
 - She told off her friends
 - She told her friends off
- If it must precede the noun phrase = IN
 - She stepped **off** the train
 - *She stepped the train **off**

Quiz [SLP2]

Find the tagging errors in the following sentences:

I/PRP need/VBP a/DT flight/NN from/IN Atlanta/NN

Does/VBZ this/DT flight/NN serve/VB dinner/NNS

I/PRP have/VB a/DT friend/NN living/VBG in/IN Denver/NNP

Can/VBP you/PRP list/VB the/DT nonstop/JJ afternoon/NN flights/NNS

Quiz [SLP2]

Find the tagging errors in the following sentences:

I/PRP need/VBP a/DT flight/NN from/IN Atlanta/NN
NNP

Does/VBZ this/DT flight/NN serve/VB dinner/NNS
NN

I/PRP have/VB a/DT friend/NN living/VBG in/IN Denver/NNP
VBP

Can/VBP you/PRP list/VB the/DT nonstop/JJ afternoon/NN flights/NNS
MD

POS Tagging Methods

Similar to text classification, we would like to use machine learning methods to do POS tagging.

Using supervised learning, we need to assemble a text corpus and manually annotate the POS for every word in the corpus (i.e., the Brown corpus) (i.e., **the corpus-based methods**).

- We can divide the corpus into training data, development data and test data

To build a good corpus

- we must define a task people can do reliably (choose a suitable POS set)
- we must provide good documentation for the task
 - so annotation can be done consistently
- we must measure human performance (through dual annotation and inter-annotator agreement)
- Often requires several iterations of refinement

The Simplest POS Tagging Method

We tag each word with its most likely part-of-speech (based on the training data)

- this works quite well: about 90% accuracy when trained and tested on similar texts
- although many words have multiple parts of speech, one POS typically dominates within a single text type

How can we take advantage of context to do better?

POS Tagger As Sequence Labeling

Sequence labeling: given a sequence of observations $x = x_1, x_2, \dots, x_n$, we need to assign a label/type/class y_i for each observation $x_i \in x$, leading to the sequence label $y = y_1, y_2, \dots, y_n$ for x ($y_i \in Y$) (Y is the set of possible POS tags)

For POS tagging, x can be an input sentence where x_i is the i -th word in the sentence, and y_i can be the POS tag of x_i in x (Y is the set of the possible POS tags in our data). E.g.,

$x =$ Does this flight serve dinner
 $y =$ VBZ DT NN VB NN

Sequence Labeling

As in text classification, we also want to estimate the distribution from the training data:

$$P(y|x) = P(y_1, y_2, \dots, y_n | x_1, x_2, \dots, x_n)$$

So, we can also obtain the predicted label sequence for x by:

$$y^* = \operatorname{argmax}_y P(y|x) = \operatorname{argmax}_y P(y_1, y_2, \dots, y_n | x_1, x_2, \dots, x_n)$$

Hidden Markov Model (HMM)

Using Bayes' Rule

$$\begin{aligned} \operatorname{argmax}_y P(y|x) &= \operatorname{argmax}_y \frac{P(x|y)P(y)}{P(x)} \\ &= \operatorname{argmax}_y P(x|y)P(y) \\ &= \operatorname{argmax}_y P(x_1, x_2, \dots, x_n | y_1, y_2, \dots, y_n) P(y_1, y_2, \dots, y_n) \end{aligned}$$

First-order Markov assumption: the probability of the label for the current step only depends on the label from the previous step, so:

$$P(y_1, y_2, \dots, y_n) = \prod_{t=1}^n P(y_t | y_{<t}) = \prod_{t=1}^n P(y_t | y_{t-1})$$

Independency assumption: the probability of the current word is only dependent on its label:

$$P(x_1, x_2, \dots, x_n | y_1, y_2, \dots, y_n) = \prod_{t=1}^n P(x_t | x_{<t}, y) = \prod_{t=1}^n P(x_t | y_t)$$

So, in HMM, we need to obtain two types of probabilities:

- The transition probabilities: $P(y_t | y_{t-1})$
- The emission probabilities: $P(x_t | y_t)$

Parameter Estimation

Using Maximum Likelihood Estimators as in Naïve Bayes (i.e., just counting):

$$P(y_t | y_{t-1}) = \frac{c(y_{t-1}, y_t)}{c(y_{t-1})}$$

How many times y_{t-1} and y_t appear together in the training data?

How many times y_{t-1} appears in the training data?

$$P(x_t | y_t) = \frac{c(x_t, y_t)}{c(y_t)}$$

How many times x_t appears with y_t in the training data?

With smoothing:

$$P(x_t | y_t) = \frac{\alpha + c(x_t, y_t)}{|Y|\alpha + c(y_t)}$$

How many probabilities we have?

Transition Probabilities

	NNP	MD	VB	JJ	NN	RB	DT
<s >	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

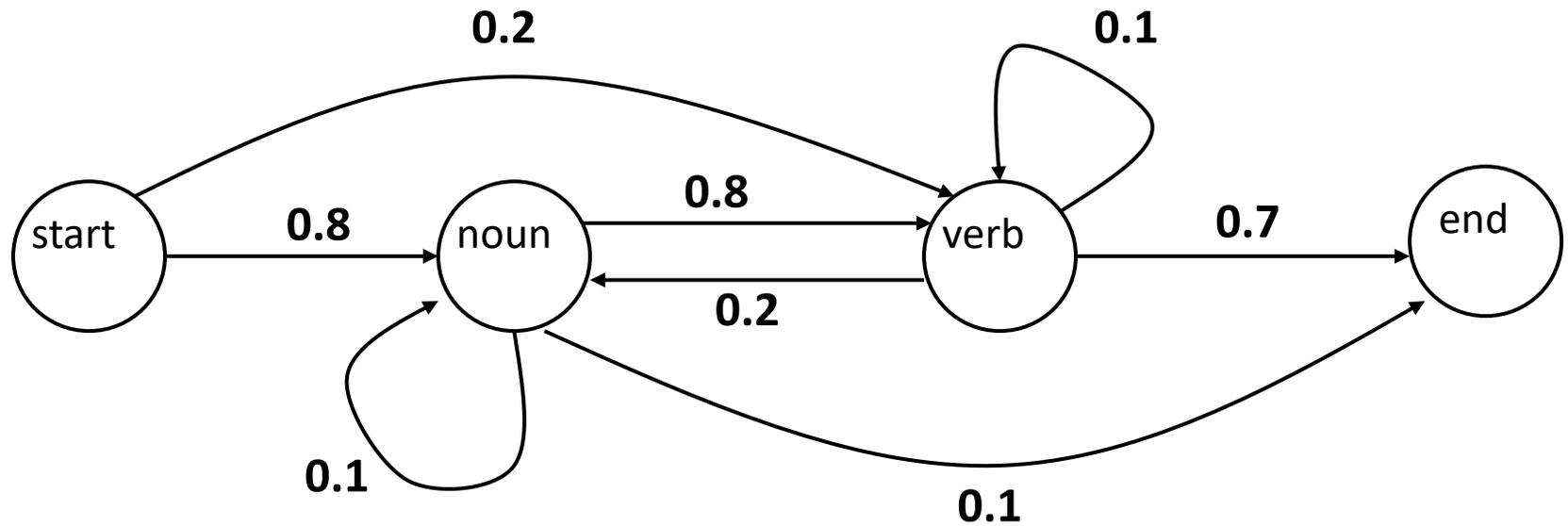
Figure 10.5 The A transition probabilities $P(t_i|t_{i-1})$ computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus $P(VB|MD)$ is 0.7968.

Emission Probabilities

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0.000097	0
NN	0	0.000200	0.000223	0.000006	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

Figure 10.6 Observation likelihoods B computed from the WSJ corpus without smoothing.

Hidden State Network



Decoding

Given the transition and emission probabilities $P(y_t|y_{t-1})$ and $P(x_t|y_t)$, we need to find the best label sequence $y^* = y_1^*, y_2^*, \dots, y_n^*$ for the input sentence $x = x_1, x_2, \dots, x_n$ via:

$$\begin{aligned}y^* &= \operatorname{argmax}_y P(y|x) \\ &= \operatorname{argmax}_y \frac{P(x,y)}{P(x)} = \operatorname{argmax}_y P(x,y) \\ &= \operatorname{argmax}_y P(x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n)\end{aligned}$$

This requires the enumeration over all the possible label sequences (paths) y which are exponentially large

- E.g., using Penn Treebank with 45 tags
 - A sentence of length 5 would have $45^5 = 184,528,125$ possible sequences
 - A sentence of length 20 would have $45^{20} = 1.16e33$ possible sequences

Greedy Decoder

simplest decoder (tagger) assign tags deterministically from left to right

selects y_t^* to maximize $P(x_t|y_t) * P(y_t|y_{t-1})$

does not take advantage of right context

can we do better?

Viterbi Algorithm

Basic idea: if an optimal path through a sequence uses label L at time t , then it must have used an optimal path to get to label L at time t

We can thus discard all non-optimal paths up to label L at time t

Let $v_t(s)$ be the probability that the HMM is in state (label) s after seeing the first t observations (words) and passing through the most probable state sequence y_1, y_2, \dots, y_{t-1} :

$$v_t(s) = \max_{y_1, y_2, \dots, y_{t-1}} P(x_1, x_2, \dots, x_t, y_1, y_2, \dots, y_{t-1}, y_t = s)$$

Introducing the *start* and *end* states to represent the beginning and the end of the sentences ($y_0 = \text{start}, y_{n+1} = \text{end}$), the probability for the optimal label sequence would be:

$$v_{n+1}(\text{end}) = \max_{y_1, y_2, \dots, y_n} P(x_1, x_2, \dots, x_n, y_0 = \text{start}, y_1, y_2, \dots, y_n, y_{n+1} = \text{end})$$

Viterbi Algorithm

$$v_t(s) = \max_{y_1, y_2, \dots, y_{t-1}} P(x_1, x_2, \dots, x_t, y_0 = \text{start}, y_1, y_2, \dots, y_{t-1}, y_t = s)$$

Initialization ($t = 0$):

$$v_0(s) = \begin{cases} 1 & \text{if } s = \text{start} \\ 0 & \text{otherwise} \end{cases}$$

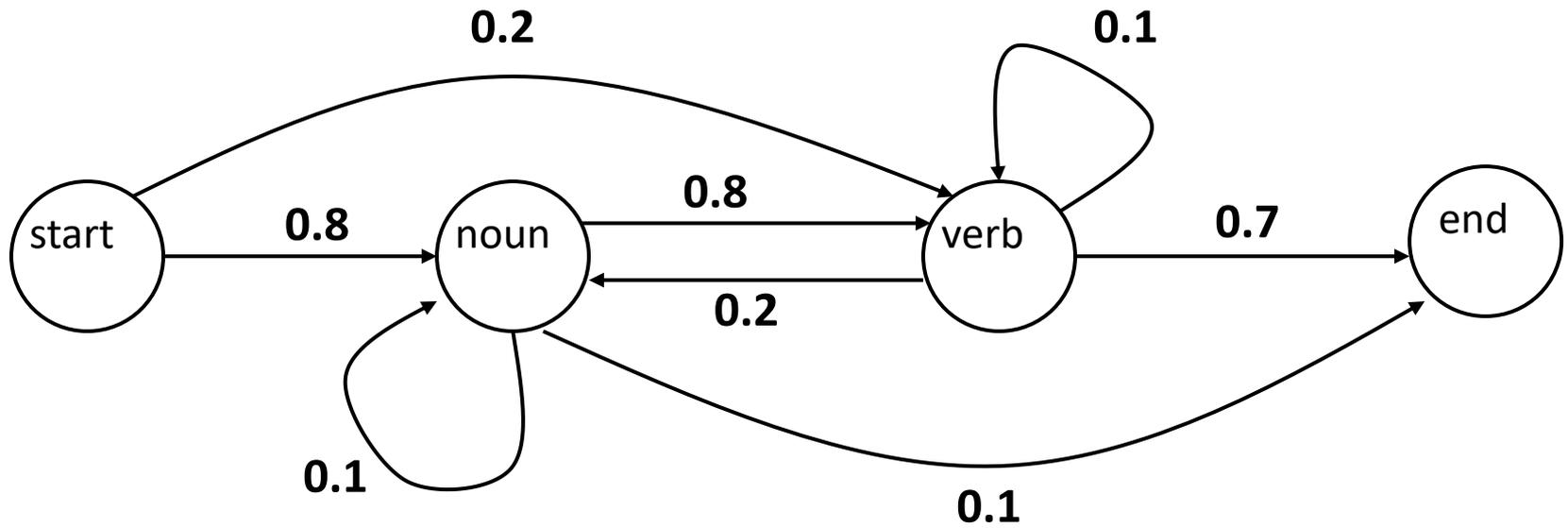
Recurrence ($t > 0$):

$$v_t(s) = \max_{s' \in Y} [v_{t-1}(s')P(s|s')P(x_t|s)]$$
$$\text{backtrack}_t(s) = \operatorname{argmax}_{s' \in Y} [v_{t-1}(s')P(s|s')P(x_t|s)]$$

Termination ($t = n + 1$): the optimal probability is $v_{n+1}(\text{end})$, following the backtrack links (starting at $\text{backtrack}_{n+1}(\text{end})$) to retrieve the optimal path.

Example

Fish sleep



Word Emission Probabilities

Word Emission Probabilities $P(\text{word} \mid \text{state})$

A two-word language: “fish” and “sleep”

Suppose in our training corpus,

- “fish” appears 8 times as a noun and 5 times as a verb
- “sleep” appears twice as a noun and 5 times as a verb

Emission probabilities:

- Noun
 - $P(\text{fish} \mid \text{noun}) : 0.8$
 - $P(\text{sleep} \mid \text{noun}) : 0.2$
- Verb
 - $P(\text{fish} \mid \text{verb}) : 0.5$
 - $P(\text{sleep} \mid \text{verb}) : 0.5$

Viterbi Probabilities

0 1 2 3

start

verb

noun

end

Noun

$P(\text{fish} \mid \text{noun}) : 0.8$

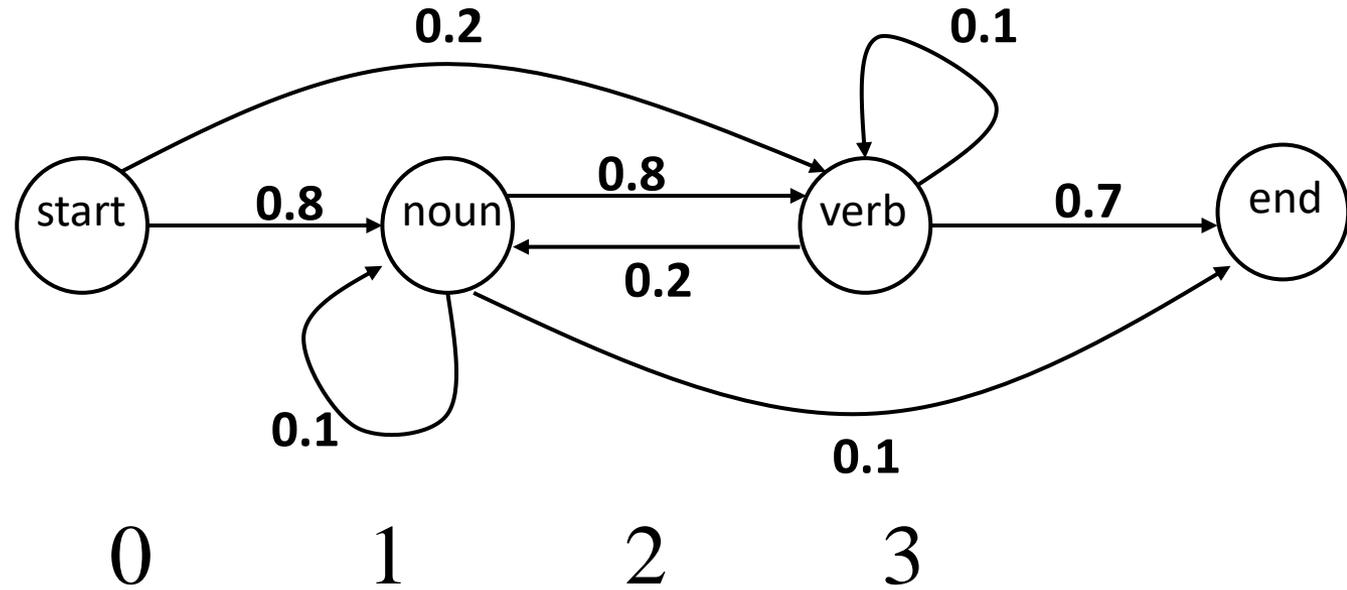
$P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$

$P(\text{sleep} \mid \text{verb}) : 0.5$

Init



	0	1	2	3
start	1			
verb	0			
noun	0			
end	0			

Noun

$P(\text{fish} \mid \text{noun}) : 0.8$

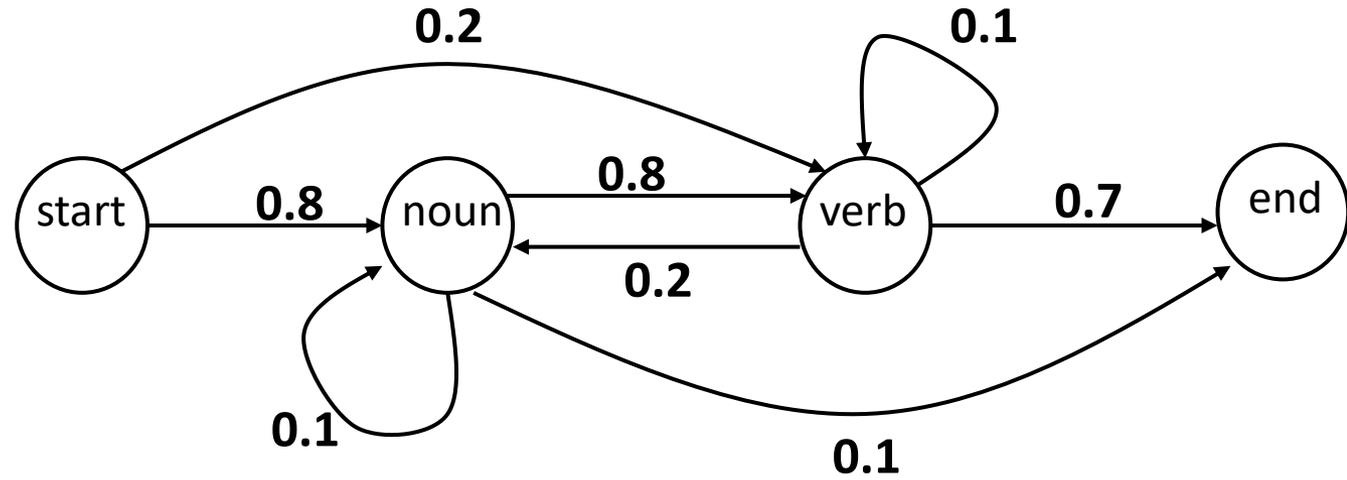
$P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$

$P(\text{sleep} \mid \text{verb}) : 0.5$

Token 1: fish



0 1 2 3

start	1	0		
verb	0	.2 * .5		
noun	0	.8 * .8		
end	0	0		

Noun

$P(\text{fish} \mid \text{noun}) : 0.8$

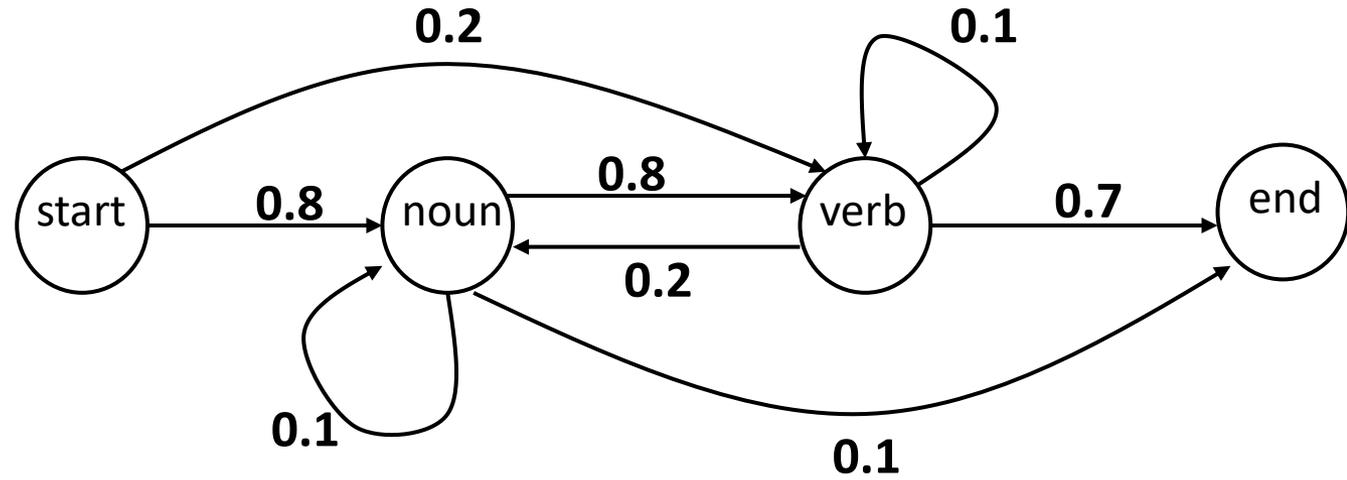
$P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$

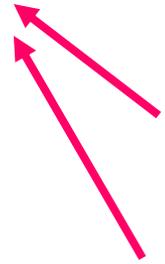
$P(\text{sleep} \mid \text{verb}) : 0.5$

Token 1: fish



0 1 2 3

start	1	0		
verb	0	.1		
noun	0	.64		
end	0	0		



Noun

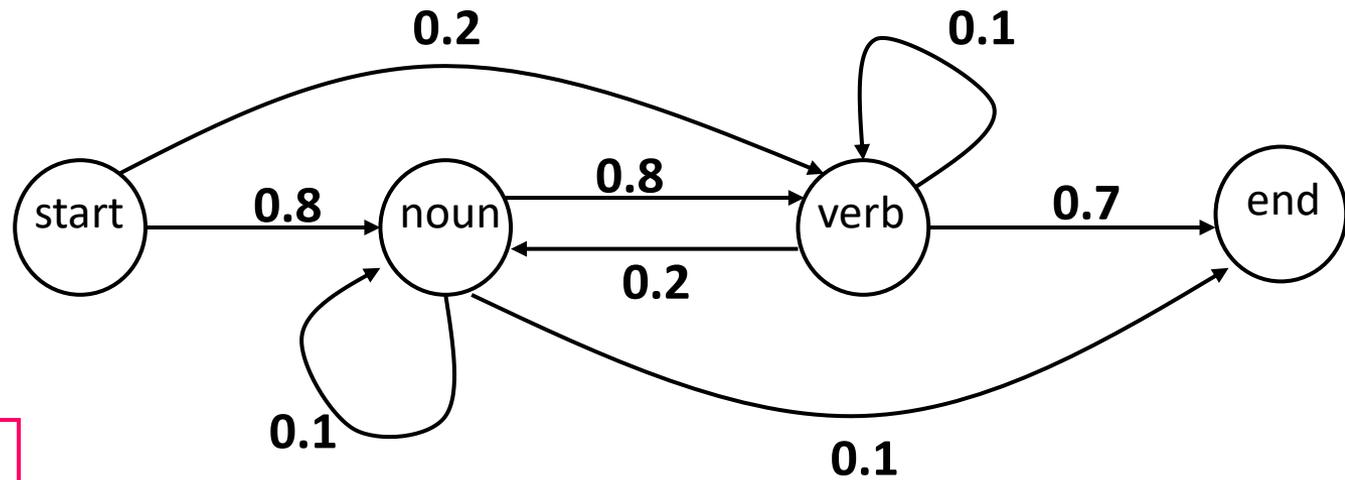
$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$

Token 2: sleep

(if 'fish' is verb)



	0	1	2	3
start	1	0	0	
verb	0	.1	.1*.1*.5	
noun	0	.64	.1*.2*.2	
end	0	0	-	

Noun

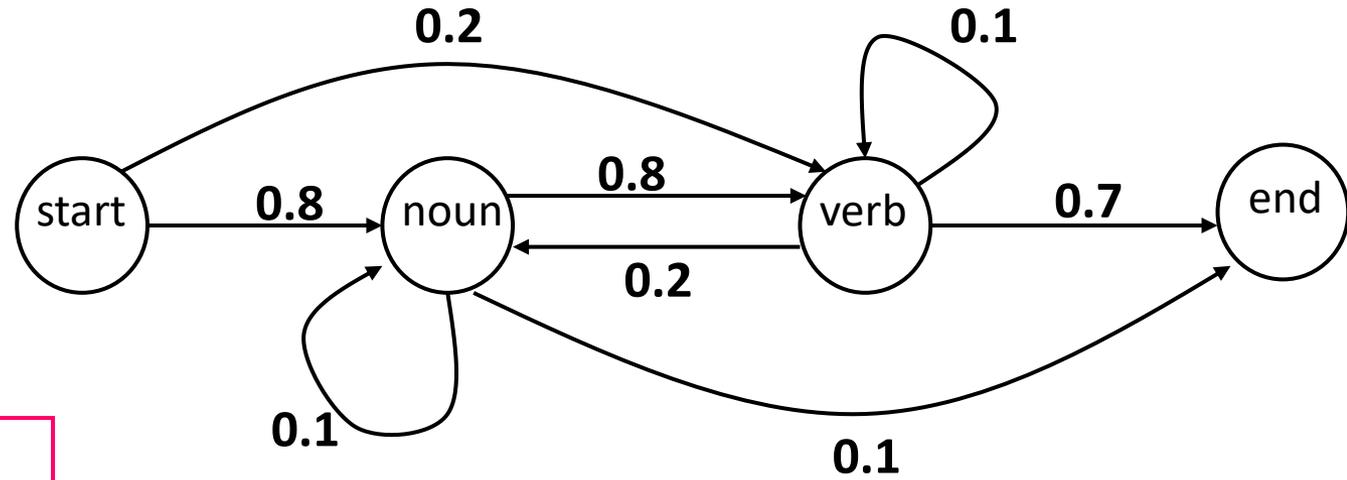
$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$

Token 2: sleep

(if 'fish' is a noun)



	0	1	2	3
start	1	0	0	
verb	0	.1	.005 $.64 * .8 * .5$	
noun	0	.64	.004 $.64 * .1 * .2$	
end	0	0	-	

Noun

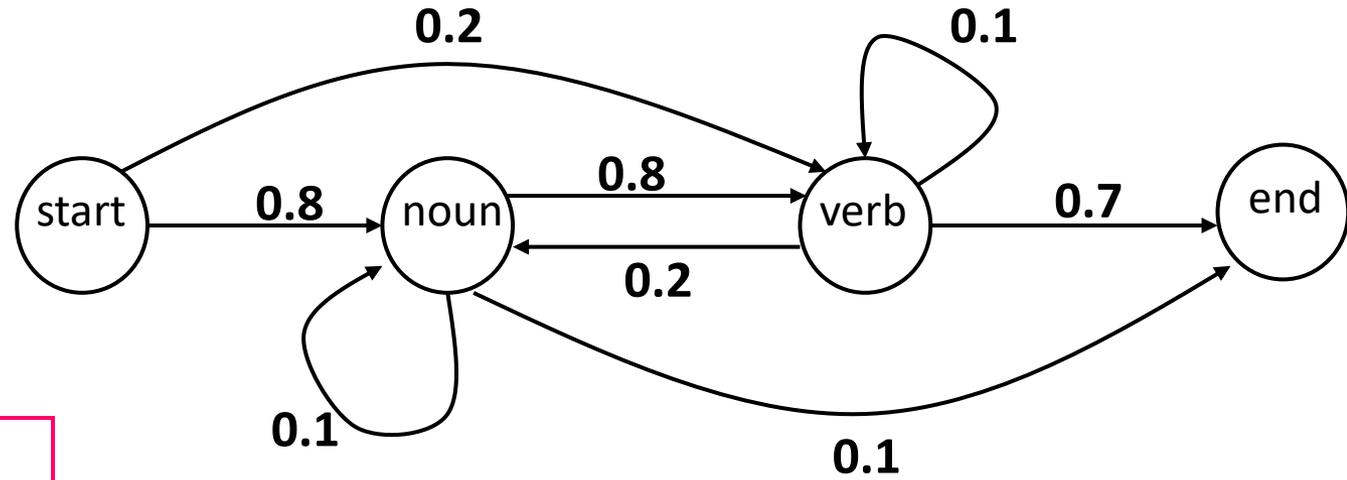
$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$

Token 2: sleep

(if 'fish' is a noun)



	0	1	2	3
start	1	0	0	
verb	0	.1	.005	.256
noun	0	.64	.004	.0128
end	0	0	-	

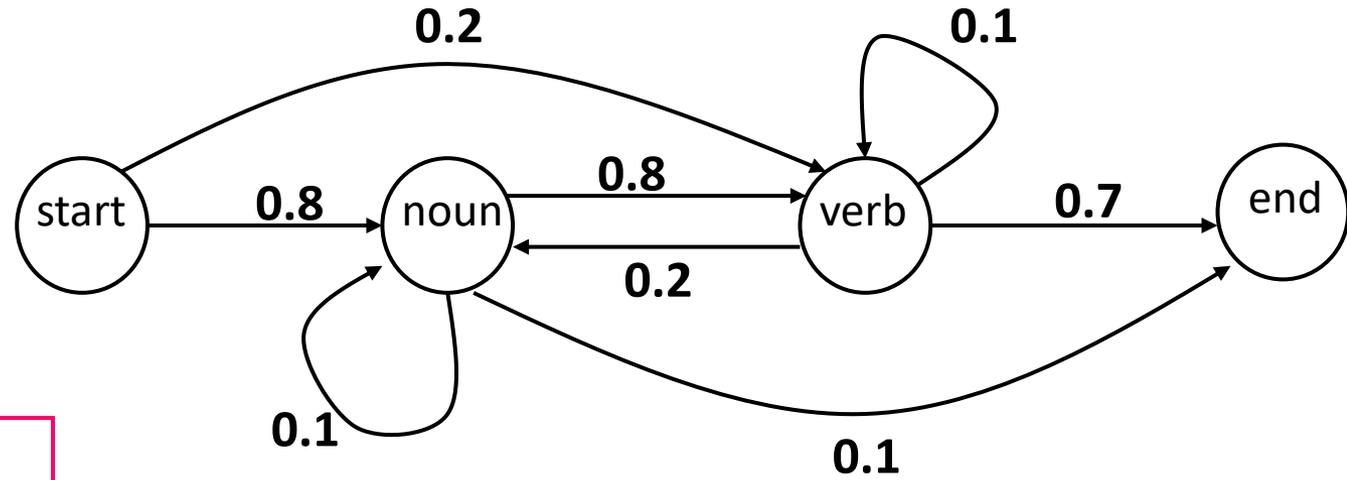
Noun

$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$

Token 2: sleep
take maximum,
set back pointers



	0	1	2	3
start	1	0	0	
verb	0	.1	.005	.256
noun	0	.64	.004	.0128
end	0	0	-	

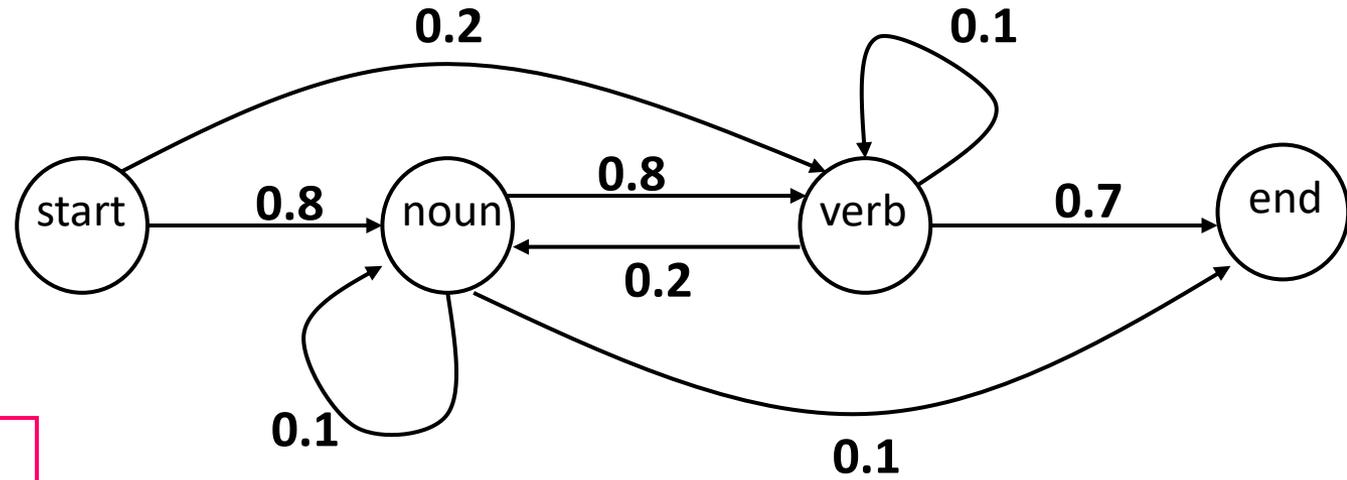
Noun

$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$

Token 2: sleep
take maximum,
set back pointers



	0	1	2	3
start	1	0	0	
verb	0	.1	.256	
noun	0	.64	.0128	
end	0	0	-	

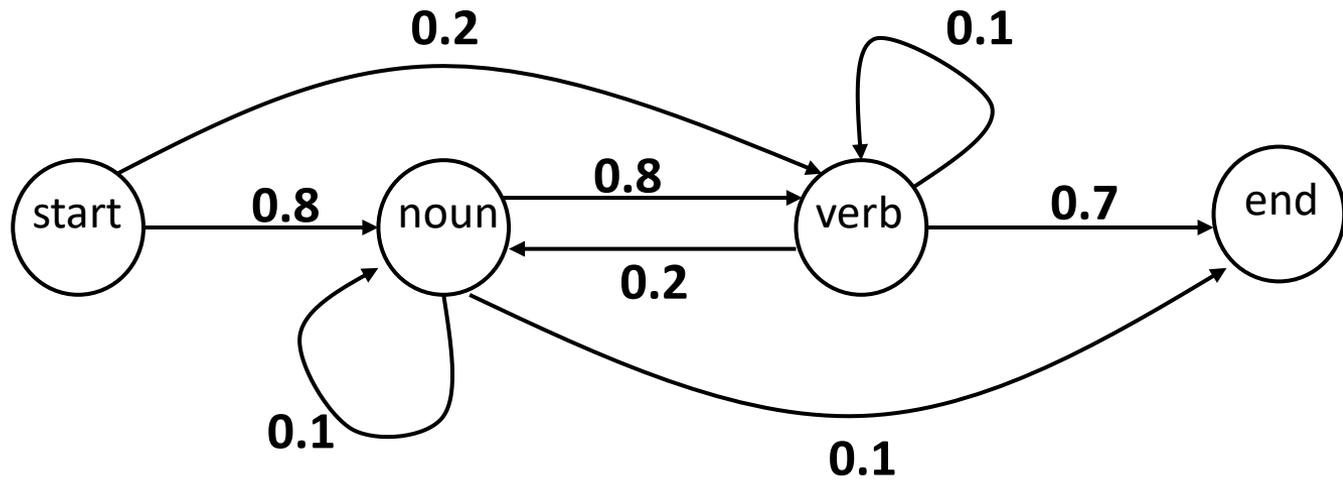
Noun

$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$

Token 3: end



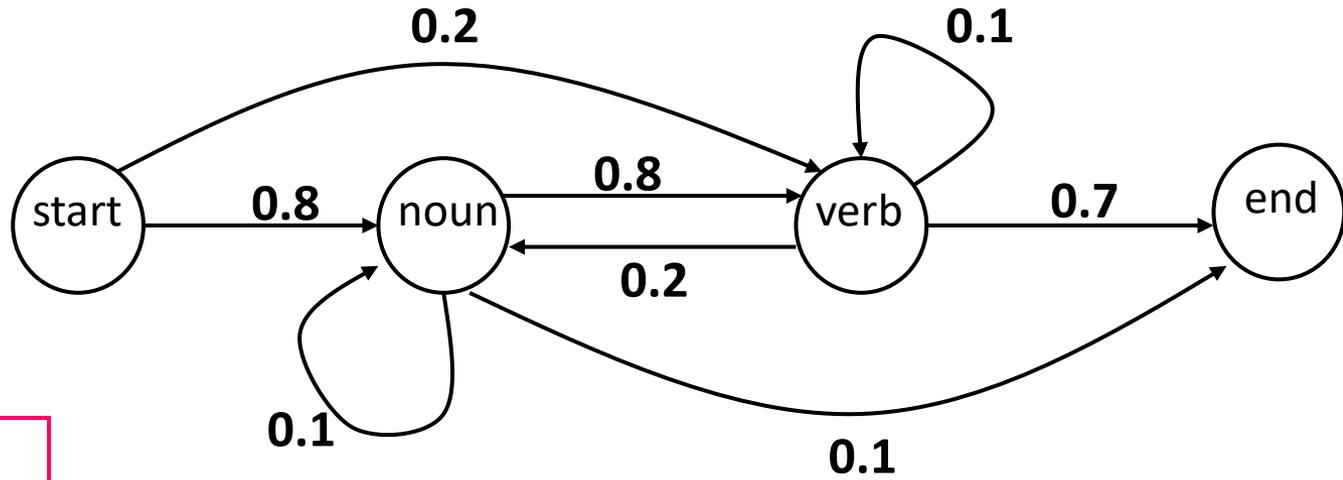
	0	1	2	3
start	1	0	0	0
verb	0	.1	.256	-
noun	0	.64	.0128	-
end	0	0	-	.256*.7 .0128*.1

Noun

$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$



Token 3: end
take maximum,
set back pointers

	0	1	2	3
start	1	0	0	0
verb	0	.1	.256	-
noun	0	.64	.0128	-
end	0	0	-	$.256 * .7$ $.0128 * .1$

Noun

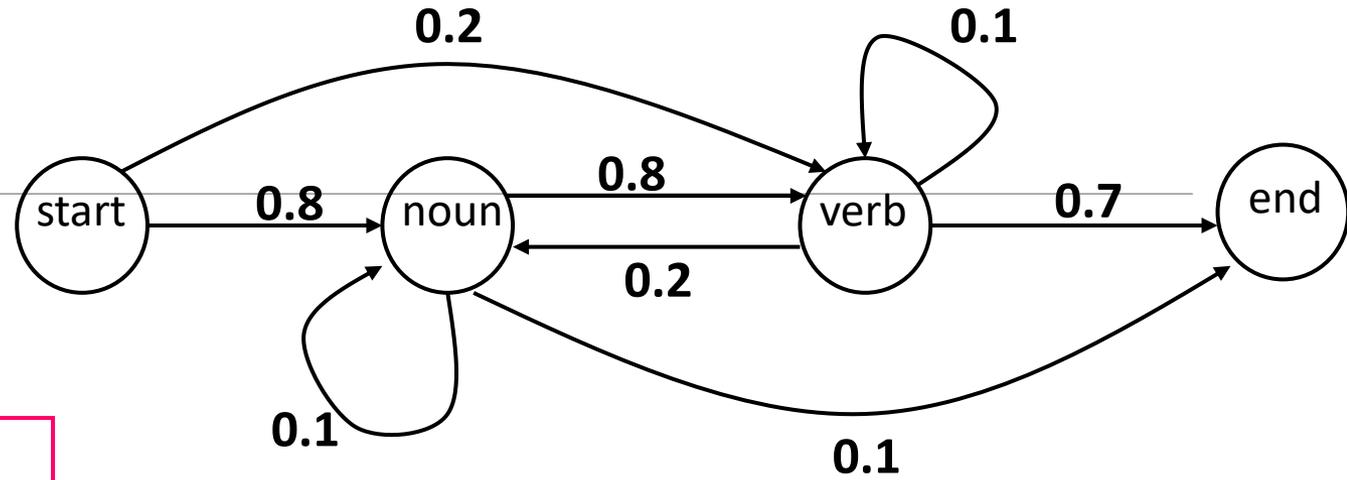
$P(\text{fish} \mid \text{noun}) : 0.8$
 $P(\text{sleep} \mid \text{noun}) : 0.2$

Verb

$P(\text{fish} \mid \text{verb}) : 0.5$
 $P(\text{sleep} \mid \text{verb}) : 0.5$

Decode:

fish = noun
sleep = verb



	0	1	2	3
start	1	0	0	0
verb	0	.1	.256	-
noun	0	.64	.0128	-
end	0	0	-	.256*.7

Complexity for Viterbi

$$\text{time} = O(s^2 n)$$

for s states (labels) and n words

(Relatively fast: for 40 states and 20 words,
32,000 steps)